EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

SECOND EXAMINATION IN SCIENCE - 2013/2014

SECOND SEMESTER (Oct./Nov.,2016)

AM 217 - MATHEMATICAL MODELING (PROPER)

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all questions **Time : Two hours**

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a) Explain the logistic model

$$\frac{dp}{dt} = ap - bp^2, \qquad p(t_0) = p_0,$$

of the population growth of a single species. Find the population $p(t)$ at time t.

b) A family of salmon fish living off the Alaskan Coast obays the Malthusian law of population growth

$$\frac{dp(t)}{dt} = 0.003p(t),$$

where $p(t)$ is the population of salmon at time t, and t is measured in minutes. At time $t = 0$, a group of sharks establishes residence in these water and begins attacking the salmon. The rate at which salmons are killed by the sharks is $0.001p^2(t)$. Moreover, since an undesirable element has moved into their neighborhood, 0.002 salmon per minute leave the Alaskan water.

i. Modify the Malthusian law of population growth to take these two factors into account.

ii. Find the population $p(t)$, by assuming at time $t = 0$ there are one million salmon. What happens as $t \to \infty$?

2. Describe the steps involved in a mathematical model building process.

Initially a 500 gallon tank contains 100 gallons of pure water. Water containing 50% pollutants flows into the tank at a rate of 2 gallons per minute and the well stirred solution is drained at a rate of 1 gallon per minute. Find the concentration of pollutants in the tank at the moment it overflows?

3. (a) Consider the D'Ancona-Voltera model for the food fish and shark community

$$\frac{dx}{dt} = ax - bxy, \quad a, b > 0;$$

$$\frac{dy}{dt} = -cy + dxy, \quad c, d > 0.$$

i. Explain the meaning of the constants appearing in the model.

ii. Show that $\frac{y^a}{e^{by}} \cdot \frac{x^c}{e^{dx}} = k$, where k is a constant.

iii. Suppose that the constant ϵ reflect the intensity of fishing rate in the above model. Discuss the modified model which incorporated the fishing effect and conclude that "reduced level of fishing is favorable to shark community than food fish".

(b) Suppose you cool a pot of soup in a $75°F$ room. When you take the soup off the stove, you measure its temperature to be $220°F$. After 20 minutes the soup has cooled to $170°F$.

i. What will be the temperature of the soup in 30 minutes.

ii. Suppose you can eat the soup when it is $130°F$, how long will it take to cool to this temperature?

a) Explain each of the terms involved in Conventional combat and Guerrilla combat.

b) A Lanchester combat model describe mixed conventional guerrilla combat (call VIETNAM) is given by

$$\begin{aligned} \frac{dx(t)}{dt} &= -a\,x(t) - g\,x(t)y(t) + P(t); \\ \frac{dy(t)}{dt} &= -d\,y(t) - c\,x(t) + Q(t). \end{aligned}$$

i. Explain the model.

ii. Suppose that no reinforcement arrive and no operational losses occur in this model(VIETNAM). Show that $gy^2(t) = 2cx(t) + (gy_0^2 - 2cx_0)$, where x_0 and y_0 are initial strengths.

iii. When do conventional forces win the combat?