EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

SECOND EXAMINATION IN SCIENCE - 2013/2014

SECOND SEMESTER (Oct./Nov., 2016)

PM 202 - METRIC SPACE

(Proper & Repeat)

Answer all questions

Time: Two Hours

1. Define what is meant by a

- *metric space*;
- *complete metric space*.

(a) Let $n \in \mathbb{N}$. The function $d : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ is defined as

$$d(x,y) = \left(\sum_{i=1}^{n} |x_i - y_i|^2 \right)^{\frac{1}{2}}, \ \forall\, x = (x_1, x_2, ..., x_n),\ y = (y_1, y_2, ..., y_n) \in \mathbb{R}^n$$

Prove that $(\mathbb{R}^n, d)$ is a complete metric space.

(b) Prove that every open ball is an open set.

(c) Prove that, for any subset A of a metric space, its interior (A°) is the largest open set contained in A.

(d) Prove that, for any subset A of a metric space, its closure $(\overline{A})$ contains the element x if and only if every open ball centered at x intersects A.

2. Define the following terms as applied to subsets of a metric space:

- *connected*;
- *separated*;
- *disconnected.*

(a) Prove that, if two connected subsets of a metric space are not separated

union is connected.

(b) Prove that two open subsets of a metric space are separated if and o

are disjoint.

(c) Prove that a metric space (X, d) is disconnected if and only if the

nonempty proper subset of X which is both open and closed.

3. Define the term *compact* as applied to subsets of a metric space.

(a) Show that every compact subset of a metric space is closed and bound

(b) Prove that every infinite subset of a compact set has a limit point.

(c) Show that $[a, b]$ is a compact subset of $\mathbb{R}$ with respect to the usual m

4. Define what is meant by a *continuous function* between two metric spaces.

(a) Let (X, d_1) and (Y, d_2) be any two metric spaces, and $f : X \to Y$ be

Prove that f is continuous at $a \in X$ if and only if for every sequence

converging to a implies that $\{f(a_n)\}$ converges to $f(a)$.

(b) Let f be a function from a metric space X into a metric space Y. Pr

is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open i

Is it true that, if f is continuous on X, then the image of an open s

open in Y? Justify your answer.

(c) Let f be a continuous function on a metric space (X, d). Prove that i

compact then $f(A)$ is compact.