EASTERN UNIVERSITY, SRI LANKA
SECOND YEAR EXAMINATION IN SCIENCE - 2012/2013
SECOND SEMESTER (Oct./Nov., 2015)
AM 218 - FIELD THEORY
(PROPER & REPEAT)

Answer all Questions **Time: Two hours**

Q1. State the *Coulomb's law* and *Gauss's law* in Electric field.

(a) A total amount of charge Q is uniformly distributed along a thin, straight, plastic rod of length L. Find the electric force acting on a point charge q located at a point P at a vertical distance y from the midpoint of the rod.

(b) A spherical volume charge density distribution is given by

$$\rho = \begin{cases} \rho_0 \left(1 - \dfrac{r^2}{a^2}\right) & r \leq a; \\ 0 & r > a, \end{cases}$$

where ρ_0 is a constant and a is the radius of the spherical volume.

(i) Calculate the total charge Q;

(ii) Find the electric field intensity E outside the sphere;

(iii) Find the electric field intensity E inside the sphere;

(iv) Show that the maximum value of E is attained at $r = 0.745a$.

Q2. (a) Define the terms *electric field strength* and *electric potential.*

(i) Suppose a very large sheet has a uniform charge density of σ coulomb per square meter. Find the electric field strength where the location of the point of intersection coincides with z-axis;

(ii) In some region of space, the electrostatic potential function, $\phi(x, y, z)$, is given by

$$\phi(x, y, z) = x^2 + 2xy + z^2,$$

where the potential is measured in volts and the distances in meters. Find the electric field at the point (2,2.2).

(b) State the *Poisson's equation* in electric field.

Show that the solution of the equation $\nabla^2\phi = 0$ in rectangular coordinates is given by

$$\phi = e^{\pm i\alpha x} e^{\pm i\beta y} e^{\pm\sqrt{\alpha^2+\beta^2}z},$$

where α and β are arbitrary constants.

Q3. (a) Using Ampere's circuit law and Biot-Savart law, prove that $\nabla^2\phi = 0$, where ϕ is scalar potential.

(b) Show that the equivalence between Biot-Savart and Ampere's laws will be brought out by determining the magnetic field $\vec{B}$ due to an infinitely long conductor carrying a steady current through it.

(c) Let a particle A of mass m_A with charge q and a particle B of mass m_B with charge $2q$ be accelerated from rest by a uniform magnetic field into semi-circular paths. If the radii of the trajectories of the particles A and B are R and $2R$, respectively and the direction of the magnetic field is perpendicular to the velocity of the particles, then show that $m_A : m_B = 1 : 8$.

Q4. (a) Define the term *magnetic flux density* and the *magnetic dipole*.

Show that $\vec{\nabla}.\vec{B} = 0$ in space, where $\vec{B}$ is a magnetic field.

(b) Find the magnetic field at the center of a current carrying square coil of a wire with sides $2a$.

(c) Let an amount of charge Q be uniformly distributed over a disk of radius R. If the disk spins about its axis with angular velocity ω, then find the magnetic dipole moment of the disk.