EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

SECOND EXAMINATION IN SCIENCE - 2012/2013

SECOND SEMESTER (Oct./Nov., 2015)

AM 217 - MATHEMATICAL MODELING

(Proper & Repeat)

Answer all questions **Time : Two hours**

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1. Describe the steps involved in a mathematical model building process.

   At 1.00 pm, Mary puts into a refrigerator a can of soda that has been sitting in a room of temperature $70°F$. The temperature in the refrigerator is $40°F$. Fifteen minutes later, at 1.15 pm, the temperature of the soda has fallen to $60°F$. At some later time, Mary removes the soda from the refrigerator to the room, where at 2.00 pm the temperature of the soda is $60°F$. At what time did Mary remove the soda from the refrigerator?

2. (a) With the usual notation, obtain the logistic law of population growth of a single species. Find the limit value of the population.

   (b) A population of bacteria grows logistically. Suppose the initial population is 3 mg of bacteria, the carrying capacity is 100 mg, and the growth parameter is 0.2 hour^{-1}.

      i. Find the differential equation satisfied by the population.

      ii. Find the population at all times.

      iii. When will the population reach 90 mg?

      iv. When will the population reach 200 mg?

3. (a) A Lanchester combat model describing a mixed conventional guerrilla combat(call VIETNAM) is given by the system of ordinary differential equations

$$\frac{dx}{dt} = -ax(t) - gx(t)y(t) + P(t),$$

$$\frac{dy}{dt} = -cx(t) - dy(t) + Q(t).$$

i. Explain the above model.

ii. Suppose that no reinforcement arrive and no operational losses occur in this model. Show that $gy^2(t) = 2cx(t) + (gy_0^2 - 2cx_0)$, where x_0 and y_0 are initial strengths.

iii. When do conventional forces win the combat?

(b) Consider the following quadratic model of two interacting species x and y:

$$\frac{dx}{dt} = x(4 - x - y); \quad \frac{dy}{dt} = y(15 - 5x - 3y).$$

i. Find the all equilibrium solutions of the model.

ii. The model have a cycle Γ defined by parametric equations $x = x(t)$, $y = y(t)$ of period T in the population quadrant. Show that the average population along Γ, $\overline{x} = \frac{1}{T}\int_0^T x(t)\,dt$ and $\overline{y} = \frac{1}{T}\int_0^T y(t)\,dt$ are given by $\overline{x} = \frac{3}{2}$ and $\overline{y} = \frac{5}{2}$.

4. (a) A tank with capacity 500 gallons initially contains 200 gallons water with 100 pounds of salt in solution. Water containing 1 pound salt per gallon enters the tank at a rate of 3 gallons per minute. The mixture flows out of the tank at a rate of 2 gallons per minute. Find the concentration of salt in the tank just before it overflows.

(b) Consider n vehicles traveling in a straight line. If $V_n(t)$ is the speed of n^{th} vehicle at time t, obtain the model

$$\frac{d}{dt}V_{n+1}(t) = V_n(t) - V_{n+1}(t).$$

Interpret this equation and show that

$$V_{n+1}(t) = \frac{1}{(n-1)!}\int_0^t u^{n-1}\, e^{-u}\, V_1(t-u)\, du$$

where $V_1(t)$ is the speed of the lead vehicle.