EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

THIRD EXAMINATION IN SCIENCE 2013/2014

FIRST SEMESTER (May/June, 2016)

AM 306 - PROBABILITY THEORY

(Proper & Repeat)

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wer all questions — Time : Two hours

culator and Statistical tables will be provided

(a) Explain briefly the "conditional probability".

i. Let A and B be two events. Show that

$$P(A \mid B^C) = \frac{P(A) - P(A \cap B)}{1 - P(B)}.$$

Hence show that $P(A \cap B) \geq P(A) + P(B) - 1$.

ii. If $P(A) \geq P(B)$ then show that $P(A \mid B) \geq P(B \mid A)$.

iii. A box of fuses contains 20 fuses of which 5 are defective. If 3 of the fuses are selected at random and removed from the box in succession without replacement. What is the probability that all three fuses are defective?

(b) i. State Bayes' theorem.

ii. Sixty percent of new drivers have had driver education. During the year, new drivers without driver education have probability 0.08 of ha an accident, but new drivers with driver education have only a 0.05 p ability of an accident. What is the probability a new driver has had d education, given that the driver has had no accident in the first year

2. (a) A random variable X has Geometric distribution with density function

$$f(x) = p\,(1-p)^{x-1}, \qquad x = 1, 2, 3, \cdots$$

where p denotes the probability of success. Find the expectation, varianc moment generating function of X.

(b) Suppose the probability of a rocket hitting a target is 0.2 and a rock repeatedly fired until the target is hit.

i. Find the expected number of rockets which will be fired.

ii. Find the probability that 4 or more rockets will be needed to final the target.

(c) Find the moment generating function of the chi-square distribution w degrees of freedom.

3. (a) If X is a random variable with density function f_X and $g(X)$ is a monoton increasing and differentiable function from $\mathbb{R}$ into $\mathbb{R}$, show that $Y = g($ the density function

$$f_Y(y) = f_X\left[g^{-1}(y)\right] \cdot \frac{d}{dy}\left(g^{-1}(y)\right), \quad \text{where } y \in \mathbb{R}.$$

If the probability density function of X is $\dfrac{e^{-x}}{(1+e^{-x})^2}$, where $-\infty < x$ then what is the probability density function of $Y = \dfrac{1}{1+e^{-x}}$.

(b) Two continuous random variable X and Y have joint density function

$$f_{XY} = cxy\ , \quad \text{where} \quad 0 < x < 4\ , \quad 1 < y < 5.$$

Find the following:

i. the value of c ;

ii. $P(X \geq 3\ , Y \leq 2)$;

iii. marginal density function of X;

iv. $P(X + Y < 3)$.

(a) Let $X_1, X_2, \cdots, X_n$ be the random samples from the normal distribution with mean μ and variance σ^2. Find the maximum likelyhood estimators μ and σ^2.

(b) Let $X_1, X_2, \cdots, X_n$ be the random samples from a population with density function

$$f(x, \theta) = \frac{1}{\theta}\, e^{-x/\theta}\ , \qquad 0 \leq x < \infty$$

where $\theta > 0$ is a parameter.

i. Are the estimators X_1 and $\overline{X} = \frac{\sum_{i=1}^{n} x_i}{n}$ unbiased for θ?

ii. Given, X_1 and $\overline{X}$, which one is more efficient estimator of θ.

(c) Let $X_1, X_2, \cdots, X_n$ be a random samples from a normal distribution with unknown mean μ and known variance σ^2. Derive the $100(1-\alpha)\%$ confidence interval for the mean μ.

(d) Let $X_1, X_2, \cdots, X_{11}$ be the random samples from a normal distribution with unknown mean μ and variance 9.9. If $\sum_{i=1}^{11} x_i = 132$, then what is the 95% confidence interval for μ?