# EASTERN UNIVERSITY, SRI LANKA
## DEPARTMENT OF MATHEMATICS
## THIRD EXAMINATION IN SCIENCE - 2012/2013
## SECOND SEMESTER (Sep./Oct., 2015)
## AM 307 - CLASSICAL MECHANICS
## (PROPER )

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Answer all Questions Time: Three hours

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1. Two frames of reference $S$ and $S'$ have a common origin $O$ and $S'$ rotates with an constant angular velocity $\underline{\omega}$ relative to $S$. If a moving particle $P$ has its position vector as $\underline{r}$ relative to $O$ at time $t$, show that :

   (a) $\dfrac{d\underline{r}}{dt} = \dfrac{\partial \underline{r}}{\partial t} + \underline{\omega} \wedge \underline{r}$, and

   (b) $\dfrac{d^2\underline{r}}{dt^2} = \dfrac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \wedge \dfrac{\partial \underline{r}}{\partial t} + \dfrac{\partial \underline{\omega}}{\partial t} \wedge \underline{r} + \underline{\omega} \wedge (\underline{\omega} \wedge \underline{r})$.

   If a projectile is fired due east from a point on the earth's surface at a northern latitude $\lambda$ with a velocity of magnitude $v_0$ and at an angle of inclination to the horizontal of $\alpha$, show that the literal deflection when the projectile strikes the earth is

$$\frac{4v_0^3}{g^2}\omega \sin\lambda \ \sin^2\alpha \ \cos\alpha,$$

   where $\omega$ is the rotation speed of the earth.

2. (a) With the usual notations obtain the *Euler's* equations for the motion of th rigid body, having a point fixed, in the form:

$$A\dot{\omega}_1 - (B - C)\omega_2\omega_3 = N_1,$$

$$B\dot{\omega}_2 - (C - A)\omega_1\omega_3 = N_2,$$

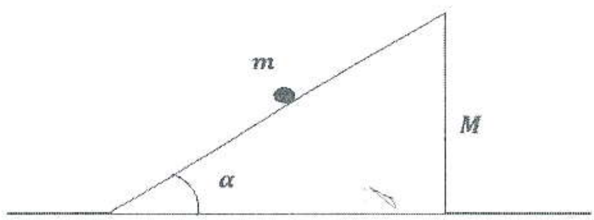
$$C\dot{\omega}_3 - (A - B)\omega_1\omega_2 = N_3.$$

A body moves about a point $O$ under no forces. The principle moment of inertia at $O$ being $3A$, $5A$ and $6A$. Initially the angular velocity has components $\omega_1 = n$, $\omega_2 = 0$, $\omega_3 = 3$ about the corresponding principal axes. Show that at time $t$,

$$\omega_2 = \frac{3n}{\sqrt{5}} \tan\left(\frac{nt}{\sqrt{5}}\right).$$

3. Obtain the *Lagrange's* equations of motion using D'Alembert's principle for a conservative holonomic dynamical system.

A point object mass $m$ is free to slide without friction down the planar surface of a wedge that is inclined at an angle $\alpha$ to the horizontal. The wedge has mass $M$ and is itself free to slide without friction on a horizontal surface (as shown in the following figure).




(a) Find the Lagrange's equations for this system subject to the force of gravity at the surface of the earth.

(b) Show that the accelerations of the object and the wedge are constant.

4. With the usual notations, derive the Lagrang's equation for the impulsive motion from the Lagrange's equations for a holonomic system in the following form

$$\triangle\left(\frac{\partial T}{\partial \dot{q}_j}\right) = S_j \text{ for } j = 1, 2, ..., n.$$

A uniform rod $AB$ of length $l$ and mass $m$ is at rest on a horizontal smooth table. An impulse of magnitude $I$ is applied to one end $A$ in the direction perpendicular to $AB$. Prove that, immediate after the application of impulse,

(a) the one end $A$ of the rod $AB$ has the velocity of magnitude $\frac{4I}{m}$,

(b) center of mass of the rod $AB$ has the velocity of magnitude $\frac{I}{m}$,

(c) the rod rotates about the center of mass with angular velocity of magnitude $\frac{6I}{m}$.

5. (a) Define *Hamiltonian* function in terms of Lagrangian function .

Show that, with the usual notations, that the Hamiltonian equations are given by

$$\dot{q_j} = \frac{\partial H}{\partial p_j}, \ \dot{p_j} = -\frac{\partial H}{\partial q_j} \text{ and } \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}.$$

(b) Prove that if the time $t$ does not occur in Lagrangian function $L$, then the Hamiltonian function $H$ is also not involved in $t$.

(c) Write down the Hamiltonian function $H$ and then find the equation of motion for a simple pendulum.

6. (a) Define the *poisson bracket.*

Show that for any function $f(q_i,\ p_i,\ t)$,

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + [f,\ H],$$

where $H$ is a Hamiltonian function.

(b) With the usual notations, prove that:

i. $\frac{\partial}{\partial t}[f,\ g] = \left[\frac{\partial f}{\partial t},\ g\right] + \left[f,\ \frac{\partial g}{\partial t}\right];$

ii. $[f,\ q_k] = -\frac{\partial f}{\partial p_k};$

iii. $[f,\ p_k] = \frac{\partial f}{\partial q_k}.$

(c) Show that, if $f$ and $g$ are constants of motion then their poisson bracket $[f,\ g]$ is also a constant of motion.