LIBRARY
17 MAR 2017
EASTERN UNIVERSITY, SRI LANKA

EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE - 2012/2013

SECOND SEMESTER (Sep./Oct., 2015)

AM 310 - FLUID MECHANICS

Answer all questions **Time : Two hours**

---

1. (a) Write the Euler's equation of motion for a fluid.

   Deduce that the Bernoulli's equation for an irrotational motion is given by $\int \frac{dp}{\rho} + \frac{1}{q^2} + \Omega =$ constant, where ρ, p, q are the density, pressure, velocity of the fluid respectively and Ω is the force potential.

   (b) A long pipe of length l has slowly tapering cross-section. It is inclined at an angle α to the horizontal and water flows steadily through it from the upper end to the lower end. The section at the upper end has twice the radius of the lower end. At the lower end the water is delivered at atmospheric pressure Π. If the pressure at the upper end is twice the atmospheric, show that the velocity of delivery is $\sqrt{\frac{32}{15}\left(gl\sin\alpha + \frac{\Pi}{\rho}\right)}$, where ρ is the density of the water.

2. (a) Let a gas occupy the region $r \leq R$, where R is a function of time, and a liquid of constant density ρ lie outside the gas. Assume that there is contact between the gas and the liquid all the time and that the motion is symmetric about the origin $r = 0$. If the velocity at $r = R$, the gas liquid boundary, is continuous, then show that the pressure p at a point $P(\underline{r}, t)$ in the liquid is given by $\frac{p}{\rho} + \frac{1}{2}\left(\frac{\dot{R}R^2}{r^2}\right)^2 - \frac{1}{r}\frac{d(\dot{R}R^2)}{dt} = f(t)$, where $r = |\underline{r}|$.

   (b) Given that a liquid extends to infinity and is at rest there with constant pressure Π. Prove that the gas and liquid interface pressure for a spherical bubble of radius R is $\Pi + \frac{\rho}{2R^2}\frac{d}{dR}(R^3\dot{R}^2)$.

(c) If the gas obeys Boyle's law, $pV^{1+\alpha} =$ constant, where α is a constant and is the volume of the gas bubble, and expands from rest at $R = a$ to a positi of rest at $R = 2a$, deduce that the initial pressure is $\frac{7\alpha\Pi}{1-2^{-3\alpha}}$.

3. (a) Let a three dimensional doublet of strength μ be situated at the origin. Sh that the velocity potential Φ at a point $P(r, \theta, \phi)$, in spherical polar coordinat due to the doublet is given by $\Phi = \mu r^{-2}\cos\theta$.

(b) Three dimensional doublets of strengths μ_1, μ_2 are situated at A and B wh cartesian coordinates are $(0, 0, a)$ and $(0, 0, b)$, their axes being directed towa and away from the origin respectively. Show that the condition for no transp of fluid across the surface of sphere $x^2 + y^2 + z^2 = ab$ is $\frac{\mu_1}{\mu_2} = \left(\frac{a}{b}\right)^{3/2}$

4. (a) Let a two dimensional source of strength m be situated at the origin. Sh that the complex potential w at a point $P(z)$ due to this source is given $w = -m \ln z$.

(b) Let the region on the positive side of the x axis be filled by a fluid of densit and the axis of y being a fixed boundary. If a two dimensional source of stren m is situated at the point $(a, 0)$, find the points on the boundary at which velocity is maximum. Show that the resultant thrust on the part of the axis y which lies between $y = \pm b$ is $2\Pi b - 2m^2\rho\left[\frac{1}{a}\tan^{-1}\left(\frac{b}{a}\right) - \frac{b}{a^2+b^2}\right]$, where is the pressure at infinity