



EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE - 2012/2013

SECOND SEMESTER (PROPER/REPEAT)

(SEPTEMBER/OCTOBER 2015)

PH 305 FUNDAMENTALS OF STATISTICAL PHYSICS

Time: 01 hour.

Answer ALL Questions

1. Derive an expression between the average thermal energy \bar{E} , and the partition function for a single particle Z , for a system of N non-interacting distinguishable particles.

A system consists of N non-interacting distinguishable identical particles, each of which can have the energy states either $+E$ or $-E$ at temperature T , where the particles do not have any translational kinetic energy.

- i. Write down an expression for partition function Z for the system.
ii. Show that the average thermal energy \bar{E} of the system is

$$\bar{E} = -\frac{(e^{\beta E} - e^{-\beta E})}{(e^{\beta E} + e^{-\beta E})}$$

- iii. Using the average thermal energy \bar{E} , in part (ii), show that the absolute temperature of the system is

$$T^{-1} = \frac{k}{2E} \ln \left[\frac{NE - U}{NE + U} \right]$$

where U is the total energy of the system.

- iv. Hence deduce an expression for the heat capacity C_V of the system.

2. State the conditions under which a system of particles obeys the Fermi-Dirac distribution law and derive an expression for the corresponding distribution.

Under which condition the distribution will reduce to the classical distribution.

Show that for a perfect gas of electron obeying Fermi-Dirac statistics, the Fermi energy of a free electron gas at absolute zero is

$$E_F = \frac{h^2}{8m} \left(\frac{3N}{\pi V} \right)^{\frac{2}{3}}$$

where the symbols have their usual meanings.

You may use the following information:

The thermodynamic probability of Fermi-Dirac distribution is;

$$\Omega = \prod_j \frac{g_j!}{(g_j - N_j)! N_j!}$$

and the number of quantum energy states between energy E and $E+dE$ is

$$g(E)dE = \frac{4\pi V(2m)^{\frac{3}{2}} E^{\frac{1}{2}}}{h^3} dE.$$