The University of Sussex

A TETRAD APPROACH TO HELICAL DEVIATION

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ABSTRACT

The aim of this thesis is to obtain a system of equations that determine the deviation between neighbouring timelike helices in the spacetime of general relativity. A helix is a curve whose principal curvatures are constant and therefore is a generalization of a geodesic, which is a curve having all its principal curvatures zero. The equations of helical deviation therefore generalise the classical equations of geodesic deviation. The form that these equations take depends on the number of non-vanishing principal curvatures and to distinguish the different cases, the terms 1-helix, 2-helix, 3-helix and 4-helix are used. The approach taken makes use of orthonormal tetrads based on a congruence of timelike curves. This is combined with a three dimensional matrix formalism that groups tetrad coefficients and curvature-tensor components into matrices and column vectors. This reduces the large number of tetrad equations to just a few matrix equations and makes them much easier to handle.

Initially the treatment is based on a general congruence of timelike curves. Then, in order to obtain the equations of helical deviation, a specialisation is made to Frenet-Serret tetrads based on a congruence of timelike helices. It is found that for a 1-helix, (i.e. a geodesic), the deviation vector satisfies a second-order system of equations, (as is well known), that for a 2-helix, (the analogue of a circle), it satisfies a third-order system while for a 3-helix, it satisfies a fourth order system and for a 4-helix, (the most general case), it satisfies a fifth-order. In order to verify and exemplify the equations obtained, examples of spacetimes that are known to possess helical congruences are considered. It is shown that for these, the deviation vector found by direct calculation satisfies the deviation equations obtained. Synge's helices in flat spacetime are also used as a check.