EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE - 1994/95 & 95/96

(Aug./Sep.'97) (REPEAT)

MT207 & 209 - CLASSICAL MECHANICS II AND DIFFERENTIAL EQUATIONS & FOURIER SERIES

Time Allowed: 02 Hours.

Answer only four questions selecting two from each section.

SECTION A

CLASSICAL MECHANICS II

1. With usual notations, obtain the following equations for a common catenary.

   (a) $s = c\tan\psi$,

   (b) $y = c\sec\psi$,

   (c) $T = \omega y$,

   (d) $y^2 = s^2 + c^2$.

   A uniform chain of length l and weight W_1 hangs between two fixed points at the same level and a weight W_2 is attached at the mid point of the chain. If the sag at the middle is d, show that the tension of the chain at each fixed point is

$$\left(\frac{d}{2l} + \frac{l}{8d}\right)W_1 + \frac{l}{4d}W_2.$$

2. State the Bernoulli-Euler law of flexure.

A beam AB of length a is clamped horizontally at A and is loaded so that the load intensity at any point P is proportional to AP^2. If the total load supported is W, then prove that the vertical force applied at B required to hold B at the same level as A is $\frac{13}{20}W$.

3. With the usual notations, prove the Claypeyron's equation

$$M_1 a + 2M_2(a+b) + M_3 b = -\frac{\omega}{4}(a^3 + b^3) + 6EI\left(\frac{y_a}{a} + \frac{y_b}{b}\right)$$

for the moment of a slightly elastic beam.

A uniform elastic beam AB of length $(a+b)$ and weight ω per unit length is clamped horizontally at its ends A & B. A support is placed at a point C in a distance 'a' from A so that A, B and C are in the same horizontal level. Prove that the bending moment of the support is

$$\frac{\omega}{12}(a^2 - ab + b^2).$$

Find the reaction at the support.

SECTION B

DIFFERENTIAL EQUATIONS & FOURIER SERIES

4. Obtain solution of the differential equation

$$x(1-x)\frac{d^2y}{dx^2} - (1+3x)\frac{dy}{dx} - y = 0$$

in series.

5. (a) State the necessary and sufficient condition for the equation $Pdx + Qdy + Rdz = 0$ to be integrable, where P, Q, R are functions of x, y and z.

   Test the integrability of the differential equation

   $$yz\, dx + (xz - yz^3)\, dy - 2xy\, dz = 0,$$

   and solve this when it is integrable.

   (b) Find the general solution of each of the following:

   i. $\dfrac{dx}{y^3x - 2x^4} = \dfrac{dy}{2y^4 - x^3y} = \dfrac{dz}{9z(x^3 - y^3)}$,

   ii. $\dfrac{dx}{x(2y^4 - z^4)} = \dfrac{dy}{y(z^4 - 2x^4)} = \dfrac{dz}{z(x^4 - y^4)}$.

   (c) Find the complete solution and singular solution of each of the following equations if $p = \dfrac{\partial z}{\partial x}$ & $q = \dfrac{\partial z}{\partial y}$.

   i. $p^2 + pq = 4z$.

   ii. $x^4p^2 - yzq - z^2 = 0$.

   (Hint: Use $X = \frac{1}{x}$, $Y = \ln y$, $Z = \ln z$.)

6\. (a) Prove that for $0 \leq x \leq \pi$,

$$\sin x = \frac{4}{\pi}\left\{\frac{1}{2} - \frac{1}{1.3}\cos 2x - \frac{1}{3.5}\cos 4x - \frac{1}{5.7}\cos 6x - \ldots\ldots\right\}.$$

(b) Define the Fourier sine transform of $F(x)$, $0 < x < \infty$, and write down its inversion formula.

Use the sine transform to show that the solution of the partial differential equation

$$\frac{\partial V}{\partial t} = 2\,\frac{\partial^2 V}{\partial x^2} \quad \text{for} \quad x > 0, \quad t > 0$$

satisfying the conditions

i. $V(0,t) = 0$,

ii. $V(x,0) = e^{-x}$,

iii. $V(x,t)$ is bounded,

is given by

$$V(x,t) = \frac{2}{\pi}\int_0^{\infty} \frac{\lambda}{1+\lambda^2}\, e^{-2\lambda^2 t} \sin \lambda x \, d\lambda.$$

(Assume that V and $\dfrac{\partial V}{\partial x}$ approach zero as $x \to \infty$).