EASTERN UNIVERSITY, SRI·LANKA

SECOND EXAMINATION IN SCIENCE - 1994/95 & 95/96

(Aug./Sep.'97) (REPEAT)

MT207 & 209 - CLASSICAL MECHANICS II AND DIFFERENTIAL EQUATIONS & FOURIER SERIES

Time Allowed: 02 Hours.

Answer only <u>four</u> questions selecting <u>two</u> from each section.

SECTION A

CLASSICAL MECHANICS II

- 1. With usual notations, obtain the following equations for a common catenary.
 - (a) $s = c \tan \psi$.
 - (b) $y = c \sec \psi$,
 - (c) $T = \omega y$,
 - (d) $y^2 = s^2 + c^2$.

A uniform chain of length l and weight W_1 hangs between two fixed points at the same level and a weight W_2 is attached at the mid point of the chain. If the sag at the middle is d, show that the tension of the chain at each fixed point is

$$\left(\frac{d}{2l} + \frac{l}{8d}\right)W_1 + \frac{l}{4d}W_2.$$

2. State the Bernoulli-Euler law of flexure.

A beam AB of length a is clamped horizontally at AB and is loaded so that the load intensity at any point P is proportional to AP^2 . If the total load supported is W, then prove that the vertical force applied at B required to hold B at the same level as A is $\frac{13}{20}W$.

3. With the usual notations, prove the Claypeyron's equation

$$M_1a + 2M_2(a+b) + M_3b = -\frac{\omega}{4}(a^3 + b^3) + 6EI\left(\frac{y_a}{a} + \frac{y_b}{b}\right)$$

for the moment of a slightly elastic beam.

A uniform elastic beam AB of length (a+b) and weight ω per unit length is clamped horizontally at its ends A & B. A support is placed at a point C in a distance 'a' from A so that A, B and C are in the same horizontal level. Prove that the bending moment of the support is

 $\frac{\omega}{12}(a^2 - ab + b^2).$

Find the reaction at the support.



DIFFERENTIAL EQUATIONS & FOURIER SERIES

4. Obtain solution of the differential equation

$$x(1-x)\frac{d^2y}{dx^2} - (1+3x)\frac{dy}{dx} - y = 0$$

in series.

5. (a) State the necessary and sufficient condition for the equation Pdx + Qdy + Rdz = 0 to be integrable, where P, Q, R are functions of x, y and z.

Test the integrability of the differential equation

$$yz \ dx + (xz - yz^3) \ dy - 2xy \ dz = 0,$$

and solve this when it is integrable.

(b) Find the general solution of each of the following:

i.
$$\frac{dx}{y^3x - 2x^4} = \frac{dy}{2y^4 - x^3y} = \frac{dz}{9z(x^3 - y^3)}$$
,

ii.
$$\frac{dx}{x(2y^4-z^4)} = \frac{dy}{y(z^4-2x^4)} = \frac{dz}{z(x^4-y^4)}$$
.

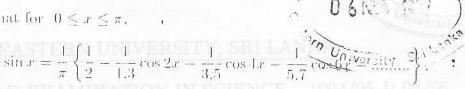
(c) Find the complete solution and singular solution of each of the following equations if $p = \frac{\partial z}{\partial x} \& q = \frac{\partial z}{\partial u}$.

i.
$$p^2 + pq = 4z$$
,

ii.
$$x^4p^2 - yzq - z^2 = 0$$
.

(Hint: Use
$$X = \frac{1}{X}$$
, $Y = \ln y$, $Z = \ln z$.)

6. (a) Prove that for
$$0 \le x \le \pi$$
,



(b) Define the Fowler sine transform of F(x), $0 < x < \infty$, and write down its pressinversion formula.

Use the sine transform to show that the solution of the partial differential equa-

$$\frac{\partial V}{\partial t} = 2 \frac{\partial^2 V}{\partial x^2} \text{ for } x > 0, \quad t > 0$$

satisfying the conditions

i.
$$V(0.t) = 0$$
,

ii.
$$V(x,0) = e^{-x}$$
,

iii.
$$V(x,t)$$
 is bounded,

is given by

$$V(x,t) = \frac{2}{\pi} \int_0^\infty \frac{\lambda}{1 + \lambda^2} e^{-2\lambda^2 t} \sin \lambda x \ d\lambda.$$

(Assume that V and $\frac{\partial V}{\partial x}$ approach zero as $x \to \infty$).