



EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
FIRST EXAMINATION IN SCIENCE -2008/2009
SECOND SEMESTER (Sep./Oct., 2010)
MT 105 - THEORY OF SERIES
(RE-REPEAT)

Answer all Questions

Time: One hour

1. (a) Define what is meant by the convergent or divergent of an infinite series $\sum_{n=1}^{\infty} a_n$.

Show that the series

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)} = \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \frac{1}{7 \cdot 9} + \dots$$

is convergent and find its sum.

- (b) Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be two series of real numbers.

i. Show that if $\sum_{n=1}^{\infty} a_n$ converges, then $a_n \rightarrow 0$ as $n \rightarrow \infty$.

ii. Is it true that, if $a_n \rightarrow 0$ as $n \rightarrow \infty$ then the series $\sum_{n=1}^{\infty} a_n$ converges?

Justify your answer.

iii. Is it true that, if $\sum_{n=1}^{\infty} (a_n + b_n)$ converges implies $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are converges? Justify your answer.

2. (a) Define the following terms:

i. absolutely convergent;

ii. conditionally convergent.

(b) i. Let $\sum_{n=1}^{\infty} a_n$ be a series of real numbers. Prove that, if $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$ also converges.

ii. Is it true that if $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} |a_n|$ converges? Justify your answer.

(c) i. Let $(a_n)_{n=1}^{\infty}$ be a decreasing sequence of positive terms such that $a_n \rightarrow 0$ as $n \rightarrow \infty$. Show that the series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ converges.

ii. Prove that the series $\sum_{n=1}^{\infty} (-1)^{n+1} \sin\left(\frac{1}{n}\right)$ converges. What will happen to this series if we drop the factor $(-1)^{n+1}$? Justify your answer.