

EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE 2001/2002

(Apr./May.'2002)

FIRST SEMESTER

Repeat

MT 203 - EIGENSPACE & QUADRATIC FORMS

Answer all questions

Time : Two hours

1. Define the term "an eigenvalue" of a linear transformation.

Explain what is meant by "a linear transformation is diagonalizable."

- (a) Prove that eigenvectors that corresponding to distinct eigenvalues of a linear transformation $T : V \rightarrow V$ are linearly independent, where V is a vector space.
- (b) Prove that if T is a linear transformation such that $T^2 = I$ then the sum of all eigenvalues of T is an integer.

Find the eigenvalues for the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ such that $T(x, y, z) = (x + 2y + 2z, x + 2y - z, -x + y + 4z)$ where $x, y, z \in \mathbb{R}$.

Further find a non-singular matrix P such that $P^{-1}AP$ is diagonal, where A is the matrix representation of T .

2. (a) Define the term "skew-symmetric" as applied to an $n \times n$ matrix.

Let A be a real skew-symmetric matrix with eigenvalue λ .

i. Prove that λ is zero or purely imaginary, and $\bar{\lambda}$ is also an eigenvalue of A .

ii. If $(A - \lambda I)^2 z = 0$ and $y = (A - \lambda I)z$ then by evaluating $(\bar{y})^t y$, show that $y = 0$, where y and z are n -column vectors.

(b) Find an orthogonal transformation which reduces the following quadratic form to a diagonal form

$$2x_1^2 + 5x_2^2 + 2x_3^2 + 4x_1x_2 + 4x_2x_3 + 2x_1x_3.$$

3. Let λ_1 and λ_2 be two distinct roots of the equation $|A - \lambda B| = 0$, where A and B are real symmetric matrices and let u_1 and u_2 be two vectors satisfying $(A - \lambda_i B)u_i = 0$ for $i = 1, 2$. Prove that $u_1^T B u_2 = 0$.

Simultaneously reduce the following pair of quadratic forms

$$\phi_1 = x_1^2 + x_2^2 + x_3^2 + 2x_2x_3 - 2x_1x_3 - 2x_1x_2$$

$$\phi_2 = 3x_1^2 + x_2^2 + 3x_3^2 - 2x_2x_3 - 2x_1x_3 + 2x_1x_2$$

4. (a) Define the minimum polynomial of a square matrix.

i. State and prove the Cayley-Hamilton theorem.

ii. Find the minimum polynomial of the matrix A given by

$$A = \begin{pmatrix} 2 & 5 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 3 & 5 \end{pmatrix}.$$

(b) State Gram-Schmidt process and use it to find the orthonormal set for span of S in \mathbb{R}^3 , where $S = \{(1, 1, 1), (0, 1, 1), (0, 0, 1)\}$.