

EASTERN UNIVERSITY SRI LANKA

DEPARTMENT OF MATHEMATICS

SPECIAL REPEAT EXAMINATION IN SCIENCE-2007/2008
THIRD YEAR, FIRST AND SECOND SEMESTER (Feb; 2010)

MT 306 - PROBABILITY THEORY

Answer all Questions

Time: Two hours

1. (a) Define the following terms:

i. Sample space;

ii. Mutually exclusive events;

(b) Define the term "conditional probability".

If A and B are two mutually exclusive events and $(A \cup B) \neq \emptyset$, using the definition of conditional probability show that

$$P(A|(A \cup B)) = \frac{P(A)}{P(A) + P(B)}$$

(c) State and prove the Baye's Theorem?

A bottle manufacturing company uses three machines A , B and C , and of their respective output 5%, 4% and 2% of the items are faulty. A manufactures 25%, B manufactures 35% and C manufactures 40% of the total output. A bottle drawn at random from the product line is found to be faulty. What is the probability that it was manufactured by B ?

2. (a) Define the term "Moment generating function" of a random variable X .
Hence show that if X and Y are independent random variables, then $X + Y$ has the Moment generating function,

$$M_{X+Y}(t) = M_X(t) M_Y(t)$$

where M_X and M_Y are moment generating function of X and Y , respectively and t is a real variable.

- (b) Let X be any continuous random variable with the probability density function $f(x)$ given by

$$f(x) = \begin{cases} x + \frac{1}{2}, & 0 \leq x \leq 1; \\ 0, & \text{otherwise.} \end{cases}$$

find the moment generating function.

- (c) If X is a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{x}{6} + \frac{1}{12}, & 0 \leq x \leq 3; \\ 0, & \text{otherwise.} \end{cases}$$

Find the probability density function $g(y)$ and the cumulative distribution function $G(y)$ of $Y = 5X + 3$.

3. (a) If $X \sim \text{Bin}(n, p)$ then prove that

(i) $E(X) = np$

(ii) $\text{Var}(X) = npq$

- (b) A pair of dice is thrown 10 times. If getting a doublet is considered a success,

find the probability of

(i) 4 success

(ii) No success

- (c) The difference between the mean and the variance of a Binomial distribution is 1 and the difference between their squares is 11. Find n ?

(d) A certain type of missile hits its target with probability $p = 0.3$. Find the number of missiles that should be fired so that there is at least a 80 percent probability of hitting the target.

(a) Random variables X and Y have joint distribution function

$$f_{XY}(x, y) = \begin{cases} c(x^2 + \frac{1}{2}xy), & 0 < x < 1, 0 < y < 1; \\ 0, & \text{otherwise.} \end{cases}$$

Find

- i. the values of c ,
 - ii. the joint distribution function of X and Y ,
 - iii. marginal density function of X .
- (b) Let the random variables X and Y have the joint probability density function

$$f_{XY}(x, y) = \begin{cases} e^{-x-y}, & \text{if } x, y > 0; \\ 0, & \text{otherwise.} \end{cases}$$

and let $U = X + Y$ and $V = \frac{X}{X + Y}$.

- i. Find the joint probability density function of U and V .
- ii. Are U and V are independent random variables?