EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

SPECIAL REPEAT EXAMINATION IN SCIENCE - 2007/2008

THIRD YEAR, FIRST AND SECOND SEMESTER (Feb., 2010)

MT 310 - FLUID MECHANICS

Answer all Questions

Time: Two hours

1. (a) Derive the continuity equation for an incompressible fluid flow in the form $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$ in cartesian coordinates, where u, v and w are the cartesian components of the velocity $\underline{q}$.

   (b) Show that $\frac{k}{r^5}(3x^2-r^2, 3xy, 3xz)$, where $r^2=x^2+y^2+z^2$ and k is a constant, represents the velocity field in a possible fluid motion.

   Show also that this motion is irrotational.

   (c) Find the velocity potential and the equation of stream lines for the velocity field given in (b).

2. (a) State the following:

   i. Milne-Thomson circle theorem,

   ii. the theorem of Blasius.

   (b) Let $\omega=(u-iv)z$ be the complex potential of an undistributed motion.

   i. If a circular cylinder is placed in the above uniform motion, find the resultant complex potential.

   ii. If the pressure thrusts on the given cylinder are represented by a force (X, Y) and a couple of moment M about the origin, where action of X

and Y are directed along the real and imaginary axis, respectively then what would be expected about the motion of the cylinder?

3. (a) Let a three dimensional doublet of strength μ be situated at the origin. Show that the velocity potential ϕ at a point $P(r, \theta, \psi)$, in spherical polar coordinates, due to the doublet can be written in the form $\phi = \mu r^{-2} \cos\theta$.

   (b) Three dimensional doublets of strength μ_1, μ_2 are situated at A_1 and A_2 whose cartesian coordinates are $(0, 0, c_1)$ and $(0, 0, c_2)$, their axes being directed towards and away from the origin respectively. Show that the condition for no transport of fluid across the surface of sphere $x^2+y^2+z^2 = c_1c_2$ is $\dfrac{\mu_2}{\mu_1} = \left(\dfrac{c_2}{c_1}\right)^{\frac{3}{2}}$.

4. (a) Suppose that a solid boundary Γ of a large spherical surface contains fluid in motion and encloses closed rigid surface S_m, $m = 1, 2, ..., k$. If fluid is at rest at infinity, prove that the kinetic energy of the moving fluid is given by

$$T = \frac{1}{2}\rho \int_V \mathrm{q}^2 dV = \frac{1}{2}\rho \sum_{m=1}^{k} \int_{Sm} \phi \frac{\partial \phi}{\partial \mathrm{n}}\, dS,$$

   where the normal n at each surface element dS being drawn outwards from the fluid and the notations given above are in usual meaning.

   (b) A solid sphere of radius a with center O is moving with uniform velocity $U\underline{i}$ in an incompressible fluid of infinite extent, which is at rest at infinity, where $\underline{i}$ is the unit vector along the axis of symmetry Ox. Suppose that a velocity potential at $P(r, \theta, \psi)$, $r \geq a$, is in the form of $\phi(r, \theta) = Ar^{-2}\cos\theta$, which satisfies the axially symmetric form of Laplace's equation in spherical polar coordinates, show that $A = \dfrac{1}{2}Ua^3$.

   Hence prove that the total kinetic energy of the sphere and fluid is given by $\dfrac{1}{2}\left(M + \dfrac{1}{2}M'\right)U^2$, where M and M' are the masses of the sphere and fluid displaced, respectively.

   Furthermore, obtain the equation of streamlines.