EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE (2001/2002)

FIRST SEMESTER

(April/May ' 2002)

Repeat

MT 207 - NUMERICAL ANALYSIS

Answer all Questions **Time : Two hours**

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1. Define:

   - absolute error;
   - relative error.

   What is meant by saying that a number $\overline{x}$ approximates x to d significant digits?

   Illustrate, with an example, the loss of significance phenomenon.

   (a) Suppose x whose actual value is 2.0 is measured as 2.05.

      i. Give the relative error occurred in measuring x.

      ii. Compute x^2, x^3, x^4 and find the relative error in each computation.

      iii. Suggest a formula for the relative error in computing x^k for a positive integer k.

(b) Compute:

i. $f(x) = x\left[\sqrt{x+1} - \sqrt{x}\right]$ at $x = 500$ to 6 significant digits accuracy;

ii. $f(x) = \dfrac{e^x - 1 - x}{x^2}$ at $x = 0.01$ to 6 significant digits accuracy;

iii. The roots of the quadratic equation

$$x^2 - 26x + 1 = 0$$

to 4 digit accuracy.

2. (a) $x = \phi(x)$ is the rearrangement of the equation $f(x) = 0$ and define the iteration,

$$x_{r+1} = \phi(x_r); \quad r = 0, 1, 2 \cdots \tag{1}$$

with the initial value x_o. If $\phi'(x)$ exist, is continuous and $|\phi'(x)| \leq$ for all x, then show that the sequence $\{x_r\}$ generated by the iteration (1) converges to the unique root α of the equation $f(x) = 0$. Following iterative methods are proposed to compute $(21)^{\frac{1}{3}}$. Investigate the convergence of the methods.

i. $x_{n+1} = \dfrac{20x_n + \dfrac{21}{(x_n)^2}}{21}$.

ii. $x_{n+1} = x_n - \dfrac{(x_n)^4 - 21x_n}{(x_n)^2 - 21}$

iii. $x_{n+1} = \left(\dfrac{21}{x_n}\right)^{\frac{1}{2}}$

(b) Define the order and the asymptotic error constant of the iteration (1) in part (a).

Show that the order of the **Newton's method** is 2 and asymptotic error constant is $\frac{1}{2}\frac{f''(\alpha)}{f'(\alpha)}$. The order of the **Secant method** is approximately 1.618. Compare the efficiency of the **Secant method** and **Newton's method**

3. Let $f(x)$ be an $(n+1)$ times continuously differentiable function of x and $f_o,\ f_1,\ \cdots,f_n$ are the values of $f(x)$ at $x = x_o,\ x_1,\ \cdots,x_n$ respectively.

(a) Derive the Lagrange's Interpolation polynomial $P_n(x)$ to estimate the value of $f(x)$ for any $x \in [x_o, x_n]$.

(b) The values of the function $f(x)$ are tabulated below:

x	0	0.5	1.5	2.0
f	0.500000	0.824361	2.240845	3.694528

Obtain a linear Interpolation polynomial and compute $f(0.75)$ and

(c) Obtain a second order interpolation polynomial and compute $f(0.75$

(d) With the usual notations, show that the error in the interpolation is given by

$$E(x) = (x - x_o)(x - x_1)\cdots(x - x_n)\frac{f^{(n+1)}(\xi)}{(n+1)!}.$$

If $\left|f^{(n+1)}(x)\right| \leq M$ in $[x_o,\ x_n]$, obtain bounds on the errors in (b) and (c).

4. Suppose you are required to compute

$$I = \int_a^b f(x)dx.$$

(a) Describe the **Trapezoidal method** to compute the value of I and derive a formula for the error. State the conditions that f should satisfy in order to apply Trapezoidal rule.

(b) With the usual notations, the **Simpson's rule** is given by

$$\int_{x_{i-1}}^{x_{i+1}} f(x)dx = \frac{h}{3}(f_{i-1} + 4f_i + f_{i+1}) - \frac{1}{90}h^5 f^{(iv)}(\eta_i), \quad \eta_i \in [x_{i-1},$$

Obtain the composite Simpson's rule and show that the composite error is

$$-\frac{1}{180}h^4(b-a)f^{(iv)}(\xi), \quad \xi \in [a,b].$$

(c) Describe **Gauss Elimination** with scaled partial pivoting. Use the following system to illustrate your answer.

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$$