EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE 2001 / 2002

(APRIL' 2002)

FIRST SEMESTER

ST 201 - STATISTICAL INFERENCE I

ANSWER ALL QUESTIONS

Time : Two Hours

Q1. (a) Define

(i) A maximum likelihood estimator

(ii) An unbiased estimator

(b) Let X be the number of successes in a binomial experiment with n trials and the probability of success p. Find the maximum likelihood estimate for p and show that it is unbiased. Derive the variance of this estimator. Is this estimator consistent? Justify your answer.

(c) A random sample of n observations $X_1, X_2, \ldots, X_n$ is taken on a random variable X which has a normal distribution with mean μ and variance σ^2. Assuming σ^2 is known, find.

(i) The method of moments estimate for μ;

(ii) The maximum likelihood estimate for μ.

Q2. A random sample $X_1, X_2, \ldots, X_n$ is taken from a poisson distribution with mean λ and it is required to estimate $\theta = \lambda^2$.

(i) Show that the sample mean, $\overline{X}$, is a sufficient statistic for θ.

(ii) Evaluate $E(\overline{X})$ and $E(\overline{X}^2)$ and hence find an unbiased estimator of θ based on $\overline{X}$.

(iii) Find the Cramer - Rao lower bound for the variance of unbiased estimators of θ.

(iv) Find the efficiency of your estimator.

Q3. (a) Describe the Neyman - Pearson approach to testing one simple hypothesis against another simple hypothesis.

(b) The number of complaints in successive weeks about a certain product are denoted by $X_1, X_2, \ldots, X_n$. These random variables are independent, Poisson with mean $\mu\theta$, where μ is known and θ is unknown. It is required to test the null hypothesis Ho: $\theta = 1$ against the alternative H_1: $\theta = 2$.

(i) A test has a critical region $\{X_1, X_2, .., X_n$ such that $\Sigma x_i > m\}$ where m has been chosen so that the test has the required significance level. Show that this is the Neyman - Pearson test.

(ii) State, with reasons whether this test is uniformly most powerful for the hypothesis Ho: $\theta = 1$ against the alternative H_1: $\theta > 1$.

(iii) Suppose that $\mu = ½$, $n = m = 2$. Find the significance level and power of the test at $\theta = 2$

Q4. (a) Define

(i) Type 1 error and

(ii) Type 11 error.

(b) A coin is tossed 5 times. Let the probability of heads at each throw be p. To test $H_0 : p = \frac{1}{2}$ against the alternative $H_1 = p = \frac{1}{2}$, the critical region is taken to be $x < 2$, where x is the number of heads obtained in the 5 throws. Find type 1 and type 11 errors and the power of the test.

(c) On the basis of the results obtained from a random sample of 100 men from a particular district, the 95% confidence interval for the mean height of the men in the district is found to be (177.22 cm, 179.18 cm). Find the value of $\bar{x}$, the mean of the sample, and σ, the standard deviation of the normal population from which the sample is drawn. Calculate the 98% confidence interval for the mean height.