EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE 2003/2004

(June/July'2005)

SECOND SEMESTER

MT 217 - MATHEMATICAL MODELING

Answer all questions

Time: Two hours

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1. Write down the steps involved in a mathematical model building process.

   Give a mathematical formulation for the following problem:

   Milk can be consumed either directly or in processed forms such as butter, yoghurt and cheese etc. The company X is the sole producer of milk in a certain region and also owns the factory to process milk into various forms. The problem for the company is to determine the relative quantities of the various products necessary to achieve this goal.

2. (a) Explain the logistic model

$$\frac{dp}{dt} = ap - bp^2, \quad p(t_0) = p_0$$

of the population growth of a single species.

Find $p(t)$ and the limiting value of $p(t)$, $t > t_0$.

(b) The population of New York city would satisfy the logistic low

$$\frac{dp}{dt} = \frac{1}{25}p - \frac{1}{25 \times 10^6}p^2$$

where t is measured in years, if we neglected the high emigration and homicide rates.

i. Modify this equation to take into account the facts that 6000 people per year move from the city and 400 people per year are murdered.

ii. Assume that the population of New York city was 8×10^6 in year 1970. Find the population for all future time. What happened at $t \to \infty$.

3. (a) At 1.00PM, Mary puts into a refrigerator a can of soda that has been sitting in a room temperature $70°F$. The temperature in the refrigerator is $40°F$. Fifteen minutes later, at 1.15PM, the temperature of the soda has fallen to $60°F$. At some later time, Mary removes the soda from the refrigerator to the room. At 2.00PM the temperature of the soda is $60°F$. At what time did Mary remove the soda from the refrigerator.

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(b) A tank initially contains 50 gallon of pure water. A salt solution
containing 2 oz/gallon of water poured into the tank at a rate
of 3 gallon/minute. The mixture is stirred and is drained out of
tank at the same rate.

   i. Find the amount of salt in the tank after 20 minutes.

   ii. Find the amount of salt in the tank after long time.

4. Suppose a x-force and y-force are engaged in combat. Conventional
combat model is given by

$$\frac{dx}{dt} = -ax(t) - by(t) + P(t)$$

$$\frac{dy}{dt} = -dy(t) - cx(t) + Q(t) .$$

Explain the terms involved in these equations.

By assuming that no reinforcement arrive and no operational losses
occur, obtain a simplified model and show that the y-force wins if the
force ratio $\dfrac{y_0}{x_0} > \sqrt{\dfrac{c}{b}}$, where $x_0 = x(0)$, $y_0 = y(0)$.

Find the strengths of the x-force and y-force at time t.