EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE (2003/2004)

SECOND SEMESTER (JUNE/JULY.'2005)

MT 218 - FIELD THEORY

Proper

Answer all questions | Time: Two hours

1. State Gauss's theorem in the electro-static field.

(a) A charge q is uniformly distributed on a circle with equations $x^2 + y^2 = a^2, z = 0$. Show, with the usual notations that the potential at the point $P(0, 0, z)$ is given by $\dfrac{q}{4\pi\epsilon_0\sqrt{a^2+z^2}}$.

Prove that the electric field at P is $\dfrac{qz}{4\pi\epsilon_0\left(a^2+z^2\right)^{\frac{3}{2}}}\underline{k}$, where $\underline{k}$ is the unit vector along z axis.

(b) A spherical conductor of radius 'a' carrying a charge q_1 is surrounded by a concentric spherical conducting sheet of radius 'b' carrying a charge q_2, both conductors being insulated. Find the potential at a point between the spheres.

If the inner conductor is connected by an insulated conducting wire, passing through a small hole in the outer conductor to a distant uncharged, insulated spherical conductor of radius 'c', prove that the later will be raised to a potential $\dfrac{q_1 b + q_2 a}{4\pi\epsilon_0 b(a+c)}$.

2. (a) Show that the equation of the lines of force of a system of N collinear point charges $q_1, q_2 \cdots, q_N$ is $\sum_{i=1}^{i=N} q_i \cos\theta_i = C$, where θ_i is the angle with the radius vectors drawn from the point charge q_i at any point of the line of force make with the line of charges and C is a constant.

(b) Let $q, -\lambda q$ be two charges placed at two points A and B respectively, where $(0 < \lambda < 1)$. Prove that the line of force separating the lines going from A to B, from those going from A to infinity leaves at angle $\cos^{-1}(1-2\lambda)$.

Prove also that if $\sqrt{\lambda} > \sin\left(\frac{\alpha}{2}\right)$ a line of force leaving A at angle α with AB will end on B at an angle $2\cos^{-1}\left(\frac{1}{\sqrt{\lambda}}\sin\left(\frac{\alpha}{2}\right)\right)$.

3. (a) A current I flows in a circular loop of wire of radius 'a'. Prove that the magnetic field at a point on the axis of the loop, at a distance z from its plane is directed along the axis and is of magnitude $\frac{Ia^2}{2(a^2+z^2)^{\frac{3}{2}}}$.

(b) A plane circuit C is defined in plane polar coordinates as

$$r = \begin{cases} a & \text{if } 0 \le \theta \le \pi \\ -\dfrac{a}{\cos\theta} & \text{if } \pi \le \theta < \dfrac{3\pi}{2} \\ \dfrac{a}{\cos\theta} & \text{if } \dfrac{3\pi}{2} < \theta \le 2\pi \end{cases}$$

Show that the magnitude of the magnetic field at the origin, when the current I flows round C in the sense θ increasing is $\frac{I(\pi+2)}{4\pi a}$.

4. (a) Derive an expression for the velocity v that a particle strikes the earth when it drops at a height h from the ground of the earth.

(b) Show that the Poisson's equation $\nabla^2 U = \dfrac{1}{r^2}\dfrac{d}{dr}\left(r^2\dfrac{dU}{dr}\right) = 4\pi\rho G$, for the gravitational potential U in a spherically symmetric distribution of matter having density ρ at a distance r from the center may be written as $\dfrac{1}{r}\dfrac{d^2}{dr^2}(rU) = 4\pi\rho G$.

A given spherical distribution of total mass M is given by,

$$\rho = \begin{cases} \rho_0 \dfrac{\sin\left(\frac{\pi r}{a}\right)}{\left(\frac{\pi r}{a}\right)} & \text{if } 0 \leq r \leq a \\ 0 & \text{if } r > a. \end{cases}$$

Show that $M = \dfrac{4\rho_0 a^3}{\pi}$.

Prove that $U = -\dfrac{GM}{a}\left(1 + \dfrac{\rho}{\rho_0}\right)$ for $r \leq a$.

Calculate the self energy of the distribution in terms of G, H and a as compared with a state of infinite diffusion.