EASTERN UNIVERSITY, SRILANKA

SECOND EXAMINATION IN SCIENCE 2003/2004

SECOND SEMESTER

(June/July, 2005)

ST-204- INFERENCE-II

Answer all questions

Time Allowed: Two hours

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1. (i) State the *Neyman-Pearson lemma* for a hypothesis test.

   (ii) What do you mean by *Uniformly most powerful test* ?

   (iii) Let $y_1, y_2, \ldots, y_{20}$ be a random sample from a normal distribution with an unknown mean of μ and known variance of $\sigma^2 = 5$. We wish to test $H_0 : \mu \leq 7$ vs $H_a : \mu > 7$.

   (a) Find the *Uniformly most powerful* test with significance level of 0.05.

   (b) For the test in part (a), find the power against each of the following alternatives: $\mu = 7.5$, $\mu = 8$, $\mu = 8.5$, $\mu = 9.0$.

   (c) Sketch the graph of the *power function*.

2. (i) For a statistical test, define the following:

   (a) *Type I error*,

   (b) *Type II error*,

   (c) *Power*,

   (d) *Critical region*,

   (e) *Significance level*.

   (ii) A manufacturer of hard safety hats for construction workers is concerned about the mean and variance of the forces helmets transmit to wearers when subjected to a standard external force. The manufacturer desires the mean force transmitted by helmets to be 800 pounds (or less), well under the legal 1000-pound limit, and σ to be less than 40. A random sample of $n = 40$ helmets was tested and sample mean and variance were found to be 825 pounds and 2350 pounds^2, respectively.

(a) If $\mu = 800$ and $\sigma = 40$, is it likely that any helmet, subjected to a standard exter force, will transmit a force to a wearer in excess of 1000 pounds?

(b) Do the data provide sufficient evidence to indicate that when subjected to the standard external force, the mean force transmitted by the helmets exceeds 800 pounds?

(c) Do the data provide sufficient evidence to indicate that σ exceeds 40?

3. (i) Define the following in the context of Decision Theory:

(a) *Action Space*,

(b) *State of nature*,

(c) *Loss function*,

(d) *Regret function*,

(ii) Mr.Rex goes to market to buy fish meat, or vegetables. Since refrigerators are not avai at his home, he wishes to buy one of these foods items sufficient for few days. His thre possible actions are buying fish, buying meat, and buying vegetables. The nature of the market in a day are high price, medium price and low price. The losses due to these act are given below. Before taking the decision, which item is to be bought, he performs a experiment that he could predict the range of price of each food item during the next fe days. That is he goes around the market and observes whether each food item is availab in plenty or in normal amount and scarcely available in the market. His estimates of the probability distribution of the data are given below:

Loss Table

	Fish	Meat	Vegetable
High	8	5	3
Medium	6	4	2
Low	4	3	1

Probability Table

	Plenty	Normal	Scarcely
High	0	0.25	0.75
Medium	0.25	0.5	0.5
Low	0.75	0.25	0

(a) List all possible strategies that Mr.Rex could take.

(b) Suppose he decides to buy fish whenever it is available in plenty, obtain the best minimax strategy that he select.

(c) Assume that he knows the prior distribution (0.25, 0.50, 0.25) of the nature of the market, then obtain the best strategy under the condition given in part (b).

(i) Suppose that we want to test the null hypothesis that the mean of a normal population with $\sigma^2 = 1$ is μ_0 against the alternative hypothesis that it is μ_1, where $\mu_1 > \mu_0$.

(a) Find the value of k such that $\bar{x} > k$ provides a critical region of size $\alpha = 0.05$ for a random sample of size n.

(b) Determine the minimum sample size needed to test the null hypothesis $\mu_0 = 10$ against the alternative hypothesis $\mu_1 = 11$ with $\beta \le 0.06$.

(ii) In a Bernoulli trial, using the number of success k in n independent trials, find the posterior distribution of p, the probability of success given k. The prior distribution of p is Uniform on $[0,1]$.

You may assume the following:

The beta function is given by $B(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,dx$

$B(\alpha,\beta) = \dfrac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ and $\Gamma(n+1) = n!$; where n is an integer.