EASTERN UNIVERSITY, SRI LANKA

FIRST EXAMINATION IN SCIENCE - 2005/2006 & 2006/2007

SECOND SEMESTER (Mar./ April., 2008)

MT 105 - THEORY OF SERIES

Proper & Repeat

Answer all questions **Time : One hour**

1. (a) Define what is meant by the infinite series $\sum_{n=1}^{\infty} a_n$ is convergent.

[5 marks]

Show that the series

$$\sum_{n=1}^{\infty} \frac{1}{(4n-1)(4n+3)} = \frac{1}{3.7} + \frac{1}{7.11} + \frac{1}{11.15} + \dots$$

is convergent and find its sum.

[30 marks]

(b) State the theorem of **Integral Test**. [10 marks]

By using the above theorem or otherwise, for the following cases of $p \in \mathbb{R}$,

i. $p > 1$,

ii. $p = 1$,

iii. $0 < p < 1$,

determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges or diverges.

[15 marks]

(c) State the theorem of **Alternating Series Test**.

[10 marks]

Use the above theorem to decide whether the following series converge or diverge:

i. $\sum_{n=1}^{\infty}(-1)^{n+1}\sin\left(\frac{1}{n}\right)$;

ii. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\,n}{(3n-1)}$. [30 marks]

2. (a) Define the followings:

i. Absolutely convergent series;

ii. Conditionally convergent series. [10 marks]

Let $\sum_{n=1}^{\infty} a_n$ be a series of real numbers (positive or negative). Prove that if $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$ converges.

[20 marks]

(b) Let $\sum_{n=1}^{\infty} a_n$ be a series which satisfies $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = l$. Prove that if $l < 1$, the series converges and if $l < 1$, the series diverges.

[30 marks]

(c) Determine whether or not the following series converge or diverge by using the above results

i. $\sum_{n=1}^{\infty}(-1)^{n+1}\sin^2\left(\frac{1}{n}\right)$,

ii. $\sum_{n=1}^{\infty}\frac{[(2n)!]^2}{(4n)!}$. [30 marks]