# EASTERN UNIVERSITY, SRI LANKA

## FIRST EXAMINATION IN SCIENCE (2003/2004)

## Nov./Dec.'2004

## FIRST SEMESTER

## Proper & Repeat

## MT 106 - TENSOR CALCULAS

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**Answer all questions**

**Time : one hour**

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1. (a) Write the transformation equation for the following tensors.

   i. $A^{ms}_{qr}$,

   ii. $B^{pqr}_{mn}$,

   iii. $C_{ijk}$.

   (b) Define the terms **symmetric** and **skew-symmetric** tensors.

   i. If $ds^2 = g_{ij}dx^i dx^j$ is an invariant, show that $g_{ij}$ is a symmetric covariant tensor of rank two.

   ii. If $A^{pq}$ and $B_{rs}$ are skew-symmetric tensors , show that $C^{pq}_{rs} = A^{pq}B_{rs}$ is a symmetric tensor.

   (c) The covariant components of a tensor in rectangular co-ordinate system are $yz,\ 3,\ 2x+y$. Find its contravariant components in cylindrical coordinates $(\rho,\ \theta,\ z)$

2. (a) Define the following:

   i. Christoffel symbols of first and second kind;

   ii. Geodesics.

(b) Determine the Christoffel symbol of second kind for the line element

$$ds^2 = (dx^1)^2 + [(x^2)^2 - (x^1)^2]\,(dx^2)^2$$

and find the corresponding Geodesic equations.

(c) With the usual notations, prove the following:

   i. $\dfrac{\partial g_{rs}}{\partial x^m} = [rm, s] + [sm, r]$,

   ii. $\dfrac{\partial g^{rs}}{\partial x^m} = -g^{rn}\Gamma^s_{mn} - g^{sn}\Gamma^p_{mn}$,

   iii. $\Gamma^p_{pq} = \dfrac{\partial\, ln\sqrt{g}}{\partial x^q}$.