EASTERN UNIVERSITY, SRILANKA

FIRST EXAMINATION IN SCIENCE 2003/04

FIRST SEMESTER (Nov/Dec' 2004)

ST 101-PROBABILITY THEORY

Answer all questions

Time: Three hours

Qu1. i) a) Prove that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Derive a equation for union of three events.

b) A problem in Statistics is given to the three students A, B and C whose chance o solving it are 0.5, 0.75 and 0.25 respectively. What is the probability that the problem will be solved if all of them try independently?

ii) a) State and prove Bayes Theorem.

b) Two methods, A and B, are available for teaching a certain industrial skill. The failure rate is 20% for A and 10% for B. However, B is more expensive and hence is used only 30% of the time (A is used the other 70%). A worker was taught the skill by one of the methods but failed to learn correctly. What is the probability that she was taught by method A?

Qu2. In an attempt to economize on her telephone bill, Rani times her calls and ensures that they never last longer than four minutes. The length of calls, T minutes, may be regarded as a random variable with probability density function:

$$f(t) = kt \; ; o < t \leq 4$$
$$= 0 \; ; \text{otherwise}$$

where k is a constant.

a) i) Show that $k = 0.125$.

ii) Find the mean and standard deviation of T.

iii) Find the probability that a call lasts between three and four minutes.

iv) What is the probability that, of five independent calls, exactly three between three and four minutes.

Calls are charged at a rate of Rs 6 per call per minute plus Rs 4 for each complete m that the calls last.

b) i) Complete the following table.

Length of call, in minutes	Probability	Cost, in Rs
0-1		
1-2		
2-3		
3-4		18

ii) Find the mean and standard deviation of the cost, in Rs, of a call.

Qu3. a) Show that a linear combination of independent normal variates is also a normal var

You may assume that the moment generating function of $N(\mu,\sigma^2)$ is $e^{\left[\mu t+\frac{1}{2}\sigma^2\right.}$

b) Scores on an examination are assumed to be normally distributed with mean 78 an variance 36.

i) What is the probability that a person taking the examination scores higher than

ii) Suppose that students scoring in the top 10% of this distribution are to receive 'A' grade. What is the minimum score a student must achieve to earn an 'A'?

iii) What must be the cut off point for passing the examination if the examiner wa only the top 28.1% of all scores to be passing?

iv) Find, approximately, what proportion of students have scores 5 or more point the score that cuts off the lowest 25%?

Qu4 a) Data files on computers have sizes measured in megabytes. When files are se one computer to another down a communication link, the number of error Poisson distribution. On average, there is one error for every 10 megabytes of d Find the probability that a 3-megabyte file is transmitted:

i) without error

ii) with two or more errors

b) Show that sum of k independent Poisson variates each with parameter $\lambda_i; i = 1,2,...k$ is also a Poisson variate.

Suppose that the number of telephone calls coming into a telephone exchange between 10 A.M. and 11 A.M. ,, say X, is a random variable with Poisson distribution with parameter 2. Similarly number of telephone calls arriving into the telephone exchange between 11 A.M. and 12 noon, say Y, has a Poisson distribution with parameter 6. If X and Y are independent, what is the probability that more than 5 calls come in between 10 A.M. and 12 noon?

u5 a) Define "moment generating function" of a random variable.

Let X be a binomial random variable with parameters n and p.

i) Show that $M_X(t) = \left(q + pe^t\right)^n$, where $q = 1 - p$

ii) Using the moment generating function find the mean and variance of X.

b) An electronic circuit contains 20 components each of which has a probability of 0.01 of being defective. Calculate the probability that:

i) none of the components is defective.

ii) two of the components are defective.

iii) at least three of the components are defective.

u6. a) The organizers of a lottery claim that the chance of winning a prize with one ticket is 1 in 54. Mr.A decides to buy a ticket each week until he wins a prize. Establish a probability model for the number of weeks required for A to win a prize. Use the model to:

i) Determine the expected number of weeks A has to wait for prize.

ii) Estimate the chance that A will win a prize in less than four weeks.

iii) Estimate the chance that A will have to wait for more than one year.

iv) Estimate the most likely number of weeks A has to wait.

c) A taxi company has 12 Ambassadors and 8 Fiats. If 5 of these taxi cabs are in the shop for repairs and ambassador is as likely to be in for repairs as a Fiat, what is the probability that,

i) 3 of them are ambassadors and 2 are Fiats?,

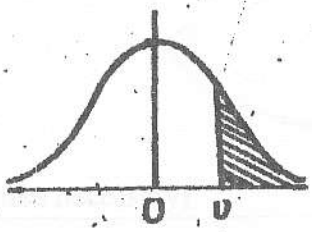
ii) at least 3 of them are ambassadors? and

iii) all of them are of the same make?

REAS OF THE STANDARDISED NORMAL DISTRIBUTION

The function tabulated is $\frac{1}{\sqrt{2\pi}}\int_u^\infty e^{-x^2/2}\,dx$,

he probability that $U > u$, where $U \sim N(0,1)$



u	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414
0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43250	0.42858	0.42465
0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
0.7	0.24196	0.23885	0.23576	0.23269	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
1.0	0.15866	0.15625	0.15386	0.15150	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08692	0.08534	0.08379	0.08226
1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003