EASTERN UNIVERSITY, SRI LANKA
THIRD EXAMINATION IN SCIENCE - 2005/2006
SECOND SEMESTER(March/April, 2008)
MT 307 - CLASSICAL MECHANICS III
(PROPER AND REPEAT)

Answer all Questions

Time: Three hours

Q1. (a) Two frame of references S and S' have a common origin O and S' rotates with angular velocity $\underline{w}$ with respect to S. At a time t, a particle P has position vector $\underline{r}$ with respect to O. With the usual notation, prove that

$$\frac{d^2\underline{r}}{dt^2} = \frac{\partial^2\underline{r}}{\partial t^2} + 2\underline{\omega} \wedge \frac{\partial \underline{r}}{\partial t} + \frac{d\underline{\omega}}{dt} \wedge \underline{r} + \underline{\omega} \wedge (\underline{\omega} \wedge \underline{r}).$$

You may use the following vector identity:

$$\frac{d(\,)}{dt} = \frac{\partial(\,)}{\partial t} + \underline{\omega} \wedge (\,).$$

[30 marks]

(b) A particle is released from rest at a height h near the earth's surface and at a latitude λ through a resisting medium with resistance $-k\frac{\partial \underline{r}}{\partial t}$, where $\frac{\partial \underline{r}}{\partial t}$ is the velocity of the particle relative to a frame of reference fixed on the earth's surface. Show that the deviation of the particle towards the East at time t is given by

$$\frac{2\omega g t(1+e^{-kt})}{k^2}\cos\lambda - \frac{4\omega g(1-e^{-kt})}{k^3}\cos\lambda.$$

[70 marks]

Q2. (a) Define the following terms:

(i) Linear momentum;

(ii) Angular momentum.

[20

(b) With the usual notations, obtain the following relations for a syste particles.

(i) $\underline{H} = \underline{r_G} \wedge M\underline{V_G} + \underline{H_G}$;

(ii) $T = T_G + \dfrac{1}{2}MV_G^2$,

where G indicates the centre of mass of the system.

[35

(c) A uniform circular disc of radius R and mass M is rigidly mounted on o of a thin light shaft CD of length L. The shaft is normal to the disc centre C. The disc rolls on a rough horizontal plane, D being fixed plane by a smooth universal joint. If the centre of the disc rotates w slipping about the vertical through D with constant angular velocity Ω the angular velocity, the kinetic energy and the angular momentum of th about D.

[45 m

Q3. (a) With the usual notation, derive the Euler's dynamical equations of moti a rigid body having a fixed point.

[35 m

(b) Using part (a), show that the kinetic energy and angular momentum o torque-free motion of a rigid body are constants.

[25 m

(c) A uniform rectangular plate of mass m with the length and breadth $2a$ $2b$, respectively, spins with constant angular velocity $\underline{\omega}$ about a diagonal. the couple which must act on the plate in order to maintain this motion.

[40 m

Q4. Consider the motion of a thin circular disc of centre O, radius r and mass m rolls on a horizontal plane. Let P be the point of contact of the disc with the p which is supposed to be rough in order to prevent slipping. Let OQ be the ver and θ be the inclination of the plane of the disc with the vertical, and let the a between a fixed horizontal direction and the tangent to the disc at P be denoted ψ. If $\underline{i}, \underline{j}$ and $\underline{k}$ are an orthogonal triad of unit vectors directed along X, Y an axes, respectively, such that $\underline{k}$ is perpendicular to the disc at its centre O and along OP,

(a) obtain the general expressions for the components of reaction on the disc at the point P of contact. [40 marks]

(b) in case of the disc rolling steadily along a circular path, with usual notation, show that the motion satisfies the differential equation

$$I_{xx}\sin\theta\cos\theta\,\frac{d^2\psi}{dt^2}-(I_{zz}+mr^2)\omega_z\cos\theta\,\frac{d\psi}{dt}-mgr\sin\theta=0.$$

[60 marks]

Q5. (a) With the usual notations, derive the Lagrange's equations

$$\Delta\left(\frac{\partial T}{\partial \dot{q}_j}\right)=\sum_i I_i\frac{\partial r_i}{\partial q_j}$$

for the impulsive motion. [40 marks]

(b) Two rods AB and BC, each of length a and mass m, are frictionlessly joined at B and lie on a frictionless table. Initially the two rods are collinear. An impulse $\underline{P}$ is applied at point A in a direction perpendicular to the line ABC. Show that the magnitudes of the velocities of the centres of mass of rods BC and AB, respectively, immediately after the impulse is applied are given by $\frac{P}{4m}$ and $\frac{5P}{4m}$. [60 marks]

Q6. (a) If f and g are two functions of the dynamical variables $\vec{q}$, $\vec{p}$ and time t, define the Poisson bracket (f,g) for f and g. If q_1 and q_2 are generalized coordinates and p_1 and p_2 are the corresponding generalized momenta, find the Poisson bracket (X,Y) when $X=q_1^2+q_2^2$ and $Y=2p_1+p_2$. [25 marks]

(b) For any function $g(q,p,t)$, show that

$$\frac{dg}{dt}=(g,H)+\frac{\partial g}{\partial t}$$

and hence, if f and g are not explicit functions of time, show that

$$(f,(g,H))+(g,(H,f))+(H,(f,g))=0,$$

where H is the Hamiltonian. [45 marks]

(c) Using the result obtained in (b) and the the equation of motion in Poisson bracket, show that, if f and g are constants through out the motion, (f,g) is also constant through out the motion. [30 marks]