EASTERN UNIVERSITY, SRI LANKA

FIRST YEAR EXAMINATION IN SCIENCE

2002/2003 & 2002/2003 (A)

SECOND SEMESTER

(April/May '2004)

Repeat

MT 105 - THEORY OF SERIES

Answer All Questions

Time: 1 Hour

Q1. (a) Define what is meant by the infinite series $\sum_{n=1}^{\infty} a_n$ is convergent.

[5 Marks]

Show that the series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+4)} = \frac{1}{1.5} + \frac{1}{2.6} + \frac{1}{3.7} + \cdots$$

is convergent and find its sum.

[30 Marks]

(b) State the theorem of **Integral Test**.

[10 Marks]

By using the above theorem or otherwise, for the following cases of $p \in \mathbb{R}$,

(i) $p > 1$,

(ii) $p = 1$,

(iii) $0 < p < 1$,

determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges or diverges.

[15 Marks]

(c) State the theorem of **Alternating Series Test**.

[10 Marks]

Use the above theorem to decide whether the following series converge or diverge:

(i) $\sum_{n=1}^{\infty} \sin\left(\frac{(n^2+1)\pi}{n}\right)$;

(ii) $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{n+1}{n(n+2)}\right)$.

[30 Marks]

2. (a) Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be series of positive real numbers such that $\left(\frac{a_n}{b_n}\right)$ tends to a finite non-zero limit as $n \longrightarrow \infty$. Prove that either $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge or diverge.

[30 Marks]

Hence determine whether the following series converge or diverge

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right).$$

[20 Marks]

(b) Define what is meant by a series, $\sum_{n=1}^{\infty} a_n$, $a_n \in \mathbb{R}$, is absolutely convergent.

[10 Marks]

Use the result that, if $\sum_{n=1}^{\infty} a_n$ is a series of real numbers such that $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$ is convergent, to show that

$$\sum_{n=1}^{\infty} (-1)^{n+1} \sin^2\left(\frac{1}{n}\right)$$

is convergent.

[40 Marks]