

EASTERN UNIVERSITY, SRI LANKA

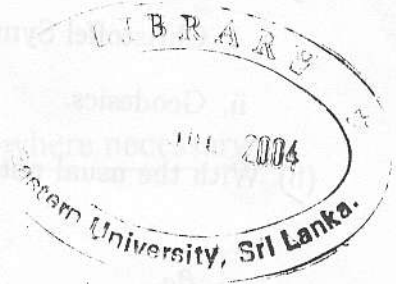
FIRST EXAMINATION IN SCIENCE-2002/03 & 2002/03(A)

(April/May-2004)

SECOND SEMESTER

Re-Repeat

MT 106 - TENSOR CALCULUS



Answer all questions

Time: One hour

1. (a) Write down the law of transformation for the following tensors:

(i) A_{qr}^{ms} , (ii) B_{mn}^{pqr} , (iii) C_{ij} .

(b) A quantity $A(p, q, r)$ is such that in the coordinate system x^t , $A(p, q, r)B_r^{qs} = C_p^s$, where B_r^{qs} is an arbitrary tensor and C_p^s is a tensor. Prove that $A(p, q, r)$ is a tensor.

(c) Prove that $A_{pq}x^p x^q = 0$ if A_{pq} is a skew-symmetric tensor.

(d) Find the covariant and contravariant components of a tensor in cylindrical coordinates (ρ, ϕ, z) if its covariant components in rectangular coordinates are $2x - z$, $x^2 y$, yz .

2. (a) Define the following:

- i. Christoffel Symbols of first and second kind,
- ii. Geodesics.

(b) With the usual notations, prove the following:

i.
$$\frac{\partial g_{pq}}{\partial x^m} = [pm, q] + [qm, p],$$

ii.
$$\frac{\partial g^{pq}}{\partial x^m} = -g^{pn}\Gamma_{mn}^q - g^{qn}\Gamma_{mn}^p;$$

iii.
$$\frac{1}{2g} \frac{\partial g}{\partial x^m} = \Gamma_{jm}^j.$$

(c) Determine the Christoffel symbol corresponding to the metric ds is given by

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

and find the corresponding Geodesic equations.