EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE (2005/2006)

FIRST SEMESTER (Aug./Sep.'2007)

MT 207 - NUMERICAL ANALYSIS

Answer all questions

Time : Two hours

1. Define "absolute error" and "relative error" of a numerical value.

(a) i. Show that the polynomial nesting technique can be used to evaluate

$$P(x) = x^3 - 3x^2 + 3x - 1.$$

ii. Evaluate the polynomial in part (i) at $x = 2.19$. Use three digit-rounding arithmetic to compute approximation to $P(2.19)$. Evaluate the absolute and relative errors.

iii. Repeat the calculation in part (ii) using the nesting form of $P(x)$ that was found in part (i). Compare the approximation with part (ii).

(b) Let $f(x) = xe^{x^2}$.

i. Find the fourth Taylor polynomial P_4 for $f(x)$ about $x_0 = 0$.

ii. Find an upper bound for the error $|f(x) - P_4(x)|$ for x in $[0, 0.4]$.

2. (a) Suppose $x = \xi$ be a unique root of a equation $f(x) = 0$, which can be rewritten as $x = \phi(x)$, contained in an interval I. Also, $x = \phi(x)$ and $x = \phi'(x)$ be continuous in I. Then, if $|\phi'(x)| \leq 1$ for all x in I, show that the iteration process defined by $x_{n-1} = \phi(x_n)$ converges to the root $x = \xi$.

Find a real root of the equation

$$f(x) = x^3 + x^2 - 1 = 0$$

by the method of iteration.

(b) Define the order and the asymptotic error constant of the iteration method to compute the nonlinear equation

$$f(x) = 0. \qquad (1)$$

i. Obtain Secant method to compute the root of the equation (1)in an interval $[a, b]$.

ii. Show that the order of convergence of Secant method is approximately 1.62.

Apply Secant method to find a solution to $x - \cos x = 0$ near $x = 0$ in the interval $[0, \frac{\pi}{2}]$ that is accurate to within 10^{-4}.

3. Suppose $x_0, x_1, ..., x_n$ are distinct numbers in the interval $[a, b]$ and $f \in C^{n+1}[a, b]$. a unique polynomial $p_n(x)$ of degree at most n with the property

$$f(x_k) = p_n(x_k) \qquad \text{for each } k = 0, 1, ..., n,$$

and show that

$$f(x) - p_n(x) = (x - x_0)(x - x_1)....(x - x_n)\frac{f^{n+1}(\xi)}{(n+1)!}$$

for each x in $[a.b]$, where $\xi(x) \in (a, b)$.

(a) A function $y = f(x)$ is give at the sample points $x = x_0, x_1$ and x_2. show that the Newton's divided interpolation formula and the corresponding Lagrange's interpolation formula are identical.

(b) Find the interpolating polynomial by (i)Lagrange's formula and (ii) Newton's divided differences formula for the following data, and hence show that they represent the same interpolating polynomial:

x	0	1	2	4
f(x)	1	1	2	5

4. (a) With the usual notations, the Simpson's rule is given by

$$\int_{x_{i-1}}^{x_{i+1}} f(x)dx = \frac{h}{3}(f_{i-1}+4f_i+f_{i+1}) - \frac{1}{90}h^5 f^{(iv)}(\xi_i) \quad \text{where } \xi_i \in [x_{i-1}, x_{i+1}].$$

Obtain the composite Simpson's rule and show that the composite error is less than or equal to

$$\frac{1}{180}h^4(b-a)|f^{iv}(\xi)|\,, \qquad \text{where } |f^{(iv)}(\xi)| = \max_{a\leq x\leq b} |f^{(iv)}(x)|.$$

A missile is launched from a ground station. The acceleration during its first 80 seconds of flight, as recorded, is given in the following table:

$t(s)$	0	10	20	30	40	50	60	70	80
$a\ (ms^{-2})$	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

compute the velocity of the missile when $t = 80\ s$, using Simpson's 1/3 rule.

(b) Describe the Gaussian Elimination with scaled partial pivoting for the solution of the equation

$$A\underline{x} = \underline{b}$$

with the usual notation.