



EASTERN UNIVERSITY, SRI LANKA
SECOND EXAMINATION IN SCIENCE - 2005/2006
(Aug./Sep.' 2007)
FIRST SEMESTER
ST 201 - STATISTICAL INFERENCE - I
(Repeat)

Answer all questions

Time : Two hours

Q1. (a) Define

- i. A maximum likelihood estimator,
- ii. An unbiased estimator.

(b) Let X be the number of success in a binomial experiment with n trials and the probability of success p . Find the maximum likelihood estimate for p and show that it is unbiased. Derive the variance of this estimator. Is this estimator consistent? Justify your answer.

(c) A random sample of n observations X_1, X_2, \dots, X_n is taken on a random variable X which has a normal distribution with mean μ and variance σ^2 . Assuming σ^2 is known, find

- i. The method of moments estimate for μ ;
- ii. The maximum likelihood estimate for μ .

Q2. A random sample X_1, X_2, \dots, X_n is taken from a poisson distribution with mean λ and it is required to estimate $\theta = \lambda^2$.

- Show that the sample mean, \bar{X} , is a sufficient statistic for θ .
- Evaluate $E(\bar{X})$ and $E(\bar{X}^2)$ and hence find an unbiased estimator of θ based on \bar{X} .
- Find the Cramer - Rao lower bound for the variance of unbiased estimators of θ .
- Find the efficiency of your estimator.

Q3. (a) Describe the Neyman - Pearson approach to testing one simple hypothesis against another simple hypothesis.

- The number of complaints in successive weeks about a certain product are denoted by X_1, X_2, \dots, X_n . These random variables are independent, Poisson with mean $\mu\theta$, where μ is known and θ is unknown. It is required to test the null hypothesis $H_0 : \theta = 1$ against the alternative $H_1 : \theta = 2$.

- A test has a critical region $\{X_1, X_2, \dots, X_n : \sum_{i=1}^n X_i > m\}$ where m is a constant to be chosen so that the test has the required significance level. Show that this is the Neyman - Pearson test.

- State, with reasons whether this test is uniformly most powerful for the hypothesis $H_0 : \theta = 1$ against the alternative $H_1 : \theta > 1$.

- Suppose that $\mu = \frac{1}{2}$, $n = m = 2$. Find the significance level and power of the test at $\theta = 2$.

Q4. (a) Define Type I error and Type II error.

- Let X_1, X_2, \dots, X_n be random samples from a normal population with parameters μ and σ^2 ($\sigma^2 = 4$).

The test is $H_0 : \mu = 0$ Vs $H_1 : \mu = 1$. The critical region is given by $\left\{ \underline{X} : \sum_{i=1}^n X_i > k \right\}$.

If $\alpha = \beta = 0.01$ then find the critical region, where

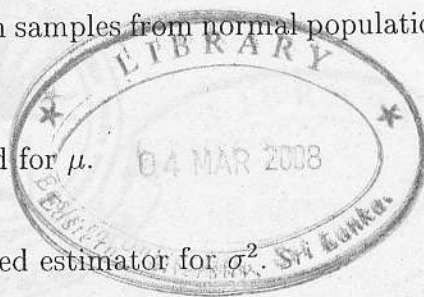
$\alpha = \text{P}(\text{Type I error})$ and $\beta = \text{P}(\text{Type II error})$

(c) Let X_1, X_2, \dots, X_n be independent random samples from normal population with mean μ and variance σ^2 . Show that,

i. the statistic $\hat{\mu} = \frac{1}{n+1} \sum_{i=1}^n X_i$ is biased for μ .

ii. $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$ is an unbiased estimator for σ^2 .

(d) Let X_1 and X_2 be independent Poisson random variables with mean m . Show that the statistic $T = X_1 - X_2$ is not sufficient.



Answer all Questions

Time Three

Q1. (a) Explain what is meant by a vector space?

Let $V = \{x \mid x \in \mathbb{R}, x > 0\}$ and let the vector addition and scalar multiplication be defined in the usual way of addition and scalar multiplication over field \mathbb{R} . Is V a vector space over \mathbb{R} ? Justify your answer. (20 marks)

(b) State the necessary and sufficient condition for a non-empty subset to be a subspace of a vector space.

Let S be a non-empty subset of a vector space V over the field F . Prove that

(i) S is the set of all linear combinations of the elements in S .

(ii) the intersection of all the subspaces containing S is the smallest subspace containing S . (15 marks)

(c) (i) Prove that $(x, y, z) = (x+y, z+x, y+z)$, for $x, y, z \in \mathbb{R}$.

(ii) Define the term 'direct sum' of two subspaces of a vector space.

Let V be the vector space of square matrices over the field \mathbb{R} . U and W are two subspaces of symmetric and anti-symmetric matrices respectively. Then show that $V = U \oplus W$. (15 marks)