EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE - 2005/2006

(Aug./Sep.' 2007)

FIRST SEMESTER

ST 201 - STATISTICAL INFERENCE - I

(Repeat)

Answer all questions

Time : Two hours

Q1. (a) Define

i. A maximum likelihood estimator,

ii. An unbiased estimator.

(b) Let X be the number of success in a binomial experiment with n trials and the probability of success p. Find the maximum likelihood estimate for p and show that it is unbiased. Derive the variance of this estimator. Is this estimator consistent? Justify your answer.

(c) A random sample of n observations $X_1, X_2, \cdots, X_n$ is taken on a random variable X which has a normal distribution with mean μ and variance σ^2. Assuming σ^2 is known, find

i. The method of moments estimate for μ;

ii. The maximum likelihood estimate for μ.

Q2. A random sample $X_1, X_2, \cdots, X_n$ is taken from a poisson distribution with mean λ and it is required to estimate $\theta = \lambda^2$.

(a) Show that the sample mean, $\bar{X}$, is a sufficient statistic for θ.

(b) Evaluate $E(\bar{X})$ and $E(\bar{X}^2)$ and hence find an unbiased estimator of θ based on $\bar{X}$.

(c) Find the Cramer - Rao lower bound for the variance of unbiased estimators of θ.

(d) Find the efficiency of your estimator.

Q3. (a) Describe the Neyman - Pearson approach to testing one simple hypothesis against another simple hypothesis.

(b) The number of complaints in successive weeks about a certain product are denoted by $X_1, X_2, \cdots, X_n$. These random variables are independent, Poisson with mean $\mu\theta$, where μ is known and θ is unknown. It is required to test the null hypothesis $H_0 : \theta = 1$ against the alternative $H_1 : \theta = 2$.

i. A test has a critical region $\{X_1, X_2, \cdots, X_n : \sum_{i=1}^{n} X_i > m\}$ where m is a constant to be chosen so that the test has the required significance level. Show that this is the Neyman - Pearson test.

ii. State, with reasons whether this test is uniformly most powerful for the hypothesis $H_0 : \theta = 1$ against the alternative $H_1 : \theta > 1$.

iii. Suppose that $\mu = \frac{1}{2}$, $n = m = 2$. Find the significance level and power of the test at $\theta = 2$.

Q4. (a) Define Type I error and Type II error.

(b) Let $X_1, X_2, \cdots, X_n$ be random samples from a normal population with parameters μ and σ^2 ($\sigma^2 = 4$).

The test is $H_0 : \mu = 0$ Vs $H_1 : \mu = 1$. The critical region is given by $\left\{ \underline{X} : \sum_{i=1}^{n} X_i > k \right\}$.

If $\alpha = \beta = 0.01$ then find the critical region, where

$\alpha =$ P(Type I error) and $\beta =$ P(Type II error)

(c) Let $X_1, X_2, \cdots, X_n$ be independent random samples from normal population with mean μ and variance σ^2. Show that,

i. the statistic $\hat{\mu} = \dfrac{1}{n+1}\sum_{i=1}^{n} X_i$ is biased for μ.

ii. $S^2 = \dfrac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is an unbiased estimator for σ^2.

(d) Let X_1 and X_2 be independent Poisson random variables with mean m. Show that the statistic $T = X_1 - X_2$ is not sufficient.