EASTERN UNIVERSITY, SRI LANKA

FIRST EXAMINATION IN SCIENCE (2005/2006 & 2006/2007)

FIRST SEMESTER (Aug./Sep.'2007)

MT 106 - TENSOR CALCULUS

Answer all questions

Time : One hour

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1. (a) Write the law of transformation for the tensors

   i. A^i_{jk},

   ii. B^{mn}_{ijk},

   iii. C^m.

   (b) Define the terms symmetric and skew-symmetric tensors.

   i. $\Phi = a_{jk}A^jA^k$ show that we can always write $\Phi = b_{jk}A^jA^k$ where b_{jk} is symmetric.

   ii. Show that the contraction of the outer product of the tensors A^p and B_q is an invariant.

   (c) Find the covariant and contravariant components of a tensor in cylindrical coordinates $(\rho,\ \phi,\ z)$, if its covariant components in rectangular coordinates are $2x - z,\ x^2y,\ yz$.

2. (a) Define the following:

   i. Christoffel symbols of first and second kind;

   ii. Geodesics;

   iii. Covariant derivative of B^i_{jk} and B_{pq}.

   (b) With the usual notations, prove the followings:

   i. $[pq, r] = [qp, r]$,

   ii. $[pq, r] = g_{rs}\Gamma^s_{pq}$,

   iii. $\dfrac{\partial g^{pq}}{\partial x^m} = -g^{pn}\Gamma^q_{mn} - g^{qn}\Gamma^p_{mn}$.

   (c) Determine the christoffel symbols of second kind in sperical coordinate (r, θ, ϕ) and find the corresponding geodesic equations.