EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
THIRD EXAMINATION IN SCIENCE - 2008/2009
SECOND SEMESTER (JANUARY, 2012)
MT 310 - FLUID MECHANICS
(SPECIAL REPEAT)

Answer all Questions — Time: Two hours

Q1. (a) Suppose that the velocity of a flow field for an incompressible fluid is

$$\underline{q} = 2x\,\underline{i} - y\,\underline{j} - z\,\underline{k}.$$

Find the streamlines passing through the point (1,1,1).

(b) Find the possible velocity components so that the surface

$$\left(\frac{x^2}{a^2}\right)\cot^2 t + \left(\frac{y^2}{b^2}\right)\sec^2 t = 1$$

to be a boundary surface for an incompressible fluid.

(c) If the velocity distribution of a fluid is

$$\underline{q} = \left(\frac{-y}{x^2+y^2}\right)\underline{i} + \left(\frac{x}{x^2+y^2}\right)\underline{j},$$

find the circulation around a square of corners (1,0), (2,0), (2,1) and (1,1).

(d) If a stream function for a two-dimensional flow of a fluid is

$$\psi = 5x - 3y + 7xy,$$

then find the velocity potential.

Q2. (a) Briefly describe the continuum hypothesis.

(b) Assume that a fluid body occupies the interior of a closed surface at tin

With the usual notation, show that the rate of change of momentum is g

by

$$\int_V \rho \frac{Du}{Dt}\, dV.$$

(c) State and prove the momentum equation for the motion of an inviscid f

Hence by considering the gravitational field, deduce the result

$$(\underline{u}.\underline{\nabla})H = 0,$$

where $H = \dfrac{P}{\rho} + \dfrac{1}{2}\,\underline{u}^2 + \chi$ and χ is a scalar potential, for a steady flow.

Q3. (a) Let a three dimensional doublet of strength μ be situated at the origin

Show that the velocity potential ϕ at a point $P(r, \theta, \psi)$, in spherical p

coordinates, due to the doublet can be written in the form of $\phi = \mu r^{-2}$ co

(b) Let three dimensional doublets of strength μ_1 and μ_2 be situated at A_1

A_2, respectively. Further, doublets are positioned at $(0, 0, c_1)$ and $(0, 0, c_2)$,

their axes being directed towards and away from the origin respectively. S

that the condition for the absence of transport of fluid across the surfac

sphere: $x^2 + y^2 + z^2 = c_1c_2$ is

$$\frac{\mu_2}{\mu_1} = \left(\frac{c_2}{c_1}\right)^{\frac{3}{2}}.$$

Q4. (a) Suppose that a solid boundary Γ of a large spherical surface contains flui

motion and encloses closed rigid surfaces S_m, $m = 1, \ldots, k$. If fluid is at

at infinity, prove that the kinetic energy of the moving fluid is given by

$$T = \frac{1}{2}\rho \int_V \mathrm{q}^2\, dV = \frac{1}{2}\rho \sum_{m=1}^{k} \int_{S_m} \phi \frac{\partial \phi}{\partial \mathrm{n}}\, dS,$$

where the normal n at each surface element dS being drawn outwards f

the fluid surface and the notations given above are in usual meaning.

(b) A solid sphere of radius a with center O is moving with uniform velocity

in an incompressible fluid of infinite extent, which is at rest at infinity, wh

$\underline{i}$ is the unit vector along the axis of symmetry Ox. Suppose that a velo

potential at $P(r, \theta, \psi)$, $r \geq a$, is in the form of

$$\phi(r, \theta) = Ar^{-2} \cos\theta,$$

which satisfies the axially symmetric form of Laplace's equation in spherical polar coordinates, show that

$$A = \frac{1}{2}Ua^3.$$

Hence prove that the total kinetic energy of the sphere and fluid is given by

$$\frac{1}{2}\left(M + \frac{1}{2}M'\right)U^2,$$

where M and M' are the masses of the sphere and fluid displaced, respectively. Furthermore, obtain the equation of streamlines.