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EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE (2004/2005)

FIRST SEMESTER (Jan./ Feb., 2006)

Repeat

MT 207 - NUMERICAL ANALYSIS

Answer all questions **Time allowed : Two hours**

1. Define "**absolute error**" and "**relative error**" of a numerical value.

   (a) Find the second Taylor polynomial $P_2(x)$ for $f(x) = e^x \cos x$ about $x_0 = 0$.

   i. Use $P_2(0.5)$ to approximate $f(0.5)$, find an upper bound for $|f(0.5) - P_2(0.5)|$, and compare this to the actual error.

   ii. Find a bound for error $|f(x) - P_2(x)|$ for x in [0,1].

   iii. Approximate $\int_0^1 f(x)\,dx$ using $\int_0^1 P_2(x)\,dx$.

   iv. Find an upper bound for the error in part (iii).

   (b) Evaluate both roots of the quadratic equation

   $$x^2 - 18x + 1 = 0$$

   as accurately assuming that only 3 significant figures can be retained in any calculations.

2. (a) Define the order and the asymptotic error constant of the iteration

$$x_{n+1} = g(x_n).$$

Show that the order of the Newton-Raphson method is 2 and asymptotic error constant is $\frac{1}{2}\frac{f''(\alpha)}{f'(\alpha)}$.

Apply Newton-Raphson method to find a Solution to $x - \cos x = 0$ in the interval $[0, \pi/2]$ that is accurate to within 10^{-4}.

(b) Let $P_n(x) = a_0x^n + a_1x^{n-1} + ... + a_n$, $a_0 \neq 0$ and let the sequence $b_0, b_1, ...b_n$ be defined by

$$b_0 = a_0$$

$$b_i = tb_{i-1} + a_i, \qquad i = 1, 2, 3...n.$$

Show that the polynomial

$$P_{n-1}(x) = b_0x^{n-1} + b_1x^{n-2} + ... + b_{n-1}$$

is the quotient polynomial and the constant $p_n(t)$ is the remainder when $p_n(x)$ is divided by $(x - t)$.

3. Let $f(x)$ be an $(n+1)$ times continuously differential function of x and $f_0, f_1, \ldots, f_n$ are the values of $f(x)$ at the distinct points $x = x_0, x_1, \ldots, x_n$ respectively.

(a) Obtain Newton's divided difference interpolation formula to estimate the value of $f(x)$ for any $x \in [x_0, x_n]$.

Find the interpolating polynomial by Newton's divided difference interpolation formula, for the following data.

x	1	2	3	5
$f(x)$	0	7	26	124

(b) With the usual notation, show that the error in the interpolation is given by

$$E_n(x) = (x - x_0)(x - x_1) \ldots (x - x_n)\frac{f^{(n+1)}(\xi)}{(n+1)!}.$$

Hence show that

$$|E_1(x)| \leq \frac{h^2 M_2}{8}$$

for the equally spaced nodes $x_k = x_0 + kh$. Where $|f^{(n+1)}(x)| \leq M_{n+1}$ for $x_0 \leq x \leq x_n$.

4. (a) Obtain Composite Trapezoidal rule to estimate $\int_a^b f(x)\,dx$ and derive a formula for the error.

   Evaluate the integral

$$\int_0^1 \frac{dx}{1+x^2}$$

   using Trapezoidal rule by taking $h = 1/4$. Hence compute the approximate value of π.

   (b) Describe Gaussian Elimination with partial pivoting for the solution of the equation

$$A\underline{x} = \underline{b}.$$

   Use the following system to illustrate your answer.

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$$