EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE (2004/2005)

FIRST SEMESTER (Jan./Feb., 2006)

ST 201 - STATISTICAL INFERENCE I

Answer all questions **Time allowed: Two hours**

1. (a) Determine the maximum likelihood estimates of the parameters for the random sample of size n from each population given below:

   i. Poisson (θ) ;

   ii. Normal (μ, σ^2) ;

   iii. Bernoulli with p .

   (b) A random sample of $X_1, X_2, ..., X_n$ is obtained from the distribution with the probability density function

$$f(x) = \frac{3\alpha^3}{(\alpha + x)^4}; \quad x > 0,$$

   where $\alpha > 0$ is the unknown parameter. Find $\hat{\alpha}$, by the method of moments.

2. Define the terms "unbiasedness" and "efficiency" in the context of estimation and state the factorization theorem.

   Let $X_1, X_2, ..., X_n$ be an independent random sample from an exponential distribution with parameter λ .

   (a) i. Find the maximum likelihood estimator for λ .

   ii. Find the sufficient statistic for λ .

   (b) Let $T_1 = \frac{2}{3}X_0^2$ where $X_0 = \frac{X_1 + X_2}{2}$ and $T_2 = \sum_{i=1}^{n} \frac{X_i^2}{2n}$

i. Show that T_1 and T_2 are unbiased estimators of $\frac{1}{\lambda^2}$.

ii. Use the efficiency method to decide which of the estimator is preferable.

3. Independent measurements $X_1, X_2, ..., X_n$ are made of a certain physical constant. The measurements are Normally distributed with mean μ and $Var(X_i) = \sigma_i^2$, where σ_i^2 is known $(i = 1, 2, ..., n)$.

(a) Find a minimal sufficient statistic for μ.

(b) Find the minimum variance unbiased estimator of μ.

(c) Show that the variance of this estimator attains the Cramer-Rao lower bound.

(d) Find the efficiency of the estimator $\bar{X}$ $(\sum X_i/n)$ relative to the minimum variance unbiased estimator.

4. The distribution of $X_{(n)}$, the largest of the n observations in a random sample from a population that is uniform on $[0, \theta]$.

(a) Show that $X_{(n)}$ is a consistent estimate of θ.

(b) Determine a multiple of $\bar{X}$ that is unbiased and obtain its mean squared error.

(c) Determine a multiple of $X_{(n)}$ that is unbiased and compute its mean squared error.

(d) What is your conclusions about these estimators?