EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE 2002/2003

SECOND SEMESTER

(April/May, 2004)

Repeat

MT306 - PROBABILITY THEORY

Answer all questions

Time : Two hours

1. (a) Let Y be a negative binomial random variable with parameters r and p and its probability mass function be given by,

$$P(Y=y)=\begin{cases}\begin{pmatrix} y-1 \\ r-1 \end{pmatrix} p^r q^{y-r} ; & y=r,r+1,r+2,\cdots, \\ 0 & \text{otherwise.}\end{cases}$$

Find

i. the expected value of Y,

ii. the variance of Y,

iii. the moment generating function of Y.

(b) The mean muscular endurance score of a random sample of 60 subjects was found to be 145 with standard deviation of 40. Construct a 95% confidence interval for the true mean. Assume the sample size to be large enough for normal approximation. What size of sample is required to estimate the mean within 5 of the true mean with a 95% confidence?

2. (a) Define Type I error, Type II error and unbiased estimator.

(b) Let $X_1, X_2, \cdots, X_n$ be random samples from a normal population with parameters μ and σ^2 ($\sigma^2 = 4$).

The test is $H_0 : \mu = 0$ Vs $H_1 : \mu = 1$. The critical region is given by $\left\{ \underline{X} \,/\, \sum_{i=1}^{n} X_i > k \right\}$. If $\alpha = \beta = 0.01$ then find the critical region, where

$\alpha = \text{P(Type I error)}$ and $\beta = \text{P(Type II error)}$

(c) Let $X_1, X_2, \cdots, X_n$ be independent random samples from normal population with mean μ and variance σ^2. Show that,

i. the statistic $\hat{\mu} = \dfrac{1}{n+1} \sum_{i=1}^{n} X_i$ is biased for μ.

ii. $S^2 = \dfrac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$ is an unbiased estimator for σ^2.

(d) Let X_1 and X_2 be independent Poisson random variables with mean m. Show that the statistic $T = X_1 - X_2$ is not sufficient.

(e) Let X and Y be independent random variables. X has the gamma distribution with parameters m and λ and Y has the gamma distribution with parameters n and λ. Show that $X + Y$ has the gamma distribution with parameters $(m + n)$ and λ.

3. (a) State the Cramer-Rao inequality.

(b) Given the probability density function,

$$f(x,\theta) = [\pi\{1+(x-\theta)^2\}]^{-1} \quad ; \quad -\infty < x < \infty, \quad -\infty < \theta < \infty.$$

show that the Cramer- Rao lower bound of variance of an unbiased estimator of θ is $\frac{2}{n}$, where n is the size of the random sample from this distribution.

4. (a) Determine the maximum likelihood estimators of the parameters of the following distributions:

i. Geometric population with parameter p.

ii. Exponential population with parameter θ.

(b) If X is a random variable having a Binomial distribution with the parameters n and θ then show that the moment generating function of $Z = \frac{X - n\theta}{\sqrt{n\theta(1-\theta)}}$ approaches that of the standard normal distribution when $n \longrightarrow \infty$.