EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE - 2004/2005

SECOND SEMESTER (Oct./ Nov., 2006)

MT 202 - ANALYSIS II (METRIC SPACE)

(Proper & Repeat)

Answer all questions   Time allowed : Two hours

---

1. Define the term *complete metric space*.

   Show that the space $C_{[-1,1]}$, the set of all real valued continuous functions on the interval $[-1, 1]$, with the function $d : C_{[-1,1]} \times C_{[-1,1]} \longrightarrow \mathbb{R}$ defined by

   $$d(f, g) = \left(\int_{-1}^{1} (f(x) - g(x))^2 \, dx\right)^{\frac{1}{2}} \quad \text{for } f, g \in C_{[-1,1]}$$

   is a metric space but not complete.

2. Let (X, d) be any metric space. Prove the following:

   (a) $a \in \bar{A} \iff \forall\, r > 0\ B(a, r) \cap A \neq \Phi$,

   (b) If D is a dense subset in X and G is an open subset of X, then $\overline{G \cap D} = \overline{G}$;

   (c) If A is dense in D and D is dense in X, then A is dense in X;

   (d) If G is open and A is disjoint from G, then $\overline{A}$ is disjoint from G.

3. Define the term *compact set* in a metric space.

   Let f be a function from a metric space (X, d_1) to a metric space (Y, d_2). Prove the following:

   (a) f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y,

(b) f is continuous if and only if $f^{-1}(B^{\circ}) \subseteq (f^{-1}(B))^{\circ} \quad \forall B \subseteq Y$,

(c) If f is continuous and A ia a compact subset of X then $f(A)$ is a compact subset in Y.

4. Define the following terms in a metric space:

- *Separated sets,*
- *Disconnected set.*

Prove the following:

(a) A metric space (X, d) is disconnected if and only if it can be written as a union of two non-empty disjoint open sets,

(b) A subset A of a metric space (X, d) is disconnected if and only if there exist open sets G_1, G_2 in (X, d) such that $G_1 \cap A \neq \Phi$, $G_2 \cap A \neq \Phi$, $G_1 \cap G_2 \cap A = \Phi$ and $A \subseteq G_1 \cup G_2$.

(c) If f is a continuous function on (X, d) and A is a connected subset of X then $f(A)$ is connected.