EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE (2004/2005)

SECOND SEMESTER (Oct./Nov.'2006)

MT 217 - MATHEMATICAL MODELING

Proper & Repeat

Answer all Questions — Time: 02 hours

1. Describe the steps involved in a mathematical model building process.

   Give a mathematical formulation for the following problem:

   A man stands at a point O and his dog at a point A in an open field. The man begins to walk at a uniform pace along a path at right angles to OA, while the dog runs at a uniform pace (greater than that of the man) always in the direction pointing to the position of the man. When does the dog catch up with the man?

2. (a) With the usual notation obtain the logistic law of population growth of a single species.

   Find the limiting value of the population and sketch the logistic curve.

   (b) Suppose a student carrying a flu virus returns to an isolated college campus of 1000 students. If it is assumed that the rate at which the virus spreads is proportional not only to the number x of infected students but also to the number of students not infected, determine the number of infected students after 6 days if it is further observed that after 4 days $x(4) = 50$.

3. Suppose a $x-$force and a y-force are engaged in combat. Let $x(t)$ and $y(t)$ denote the respective strength of the forces at time t, when t is measured in days from the start of the combat. conventional combat model is given by

$$\frac{d}{dt}x(t) = -ax(t) - by(t) + P(t)$$

$$\frac{d}{dt}y(t) = -dy(t) - cx(t) + Q(t).$$

Explain the terms involved in these equations.

By assuming that there is no reinforcement arrived and no operational losses occur, obtain a simplified model and hence show that

$$x(t) = x_0 \cosh(\beta t) - \gamma y_0 \sinh(\beta t),$$

where $\beta = \sqrt{bc}$, $\gamma = \sqrt{b/c}$ and x_0, y_0 are the initial strength of the respective forces.

4. (a) The fish population in a certain part of the sea can be separated into prey population(food fish) $x(t)$ and predator population (Selachians) $y(t)$. The model governing the the interaction of the Selachians and food fish in the absence of fishing is given by

$$\frac{d}{dt}x(t) = ax(t) - bx(t)y(t)$$

$$\frac{d}{dt}y(t) = -cy(t) + dx(t)y(t).$$

Show that

$$\frac{y^a}{e^{by}} \cdot \frac{x^c}{e^{dx}} = k,$$

where k is a constant.

(b) State the Newton's law of cooling.

At 1.00 pm Marry puts in to a refrigerator a can of soda that has been sitting in a room of temperature 70^0 F. The temperature in the refrigerator is 40^0 F. Fifteen minutes later, at 1.15 pm; the temperature of the soda has fallen to 60^0 F. At some later time Marry removes the soda from the refrigerator to the room, where at 2.00 pm the temperature of the soda is 60^0 F. At what time did Marry remove the soda from the refrigerator?