EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE(2004/05)

SECOND SEMESTER (Oct./Nov.'2006)

MT 218 - FIELD THEORY

(Proper and Repeat)

Answer all questions

Time: Two hours

1. State Gauss's theorem in the electro-static field.

   (a) A long cylinder carries a charge density $\rho = Ar$ which is proportional to the distance r from the axis, where A is a constant. Find the electric field inside the cylinder as a function of r.

   (b) Find the potential along the axis of a disk of charge density σ and radius b. Hence from this potential function find the electric field along the axis.

   (c) Three charges, each of q, are placed at three corners of a square of size d. How much work is done to bring another charge $-3q$ from far away to place it at the fourth corner? How much work is required to assemble the whole configuration of four charges?

2. (a) A line of charge has a charge density λ and is $2L$ long. Find the electric field at a distance x from the centre and at right angel to the line of charge.

   (b) A semi-infinite sheet of charge density σ is described by $-\infty < x < 0, -\infty < y < \infty$ in the $z = 0$ plane. Calculate the component of electric field normal to the sheet at a distance z directly above the edge at $x = 0$.

(c) An infinite sheet of uniform charge density σ lying in the $z = 0$ plane with a circular hole of radius a centered at the origin cut from it. Prove that the electric field along the $z-$axis is given by $\frac{\sigma z}{2\epsilon_0(z^2+a^2)^{\frac{1}{2}}}$, where ϵ_0 permitivity of the free space.

3. (a) A current I flows in a wire in the form of a part of the curve $2\pi r = a\theta$, $0 \leq \theta \leq 2\pi$. Prove that the component of the magnetic field at a point distance z from O (origin of the coordinate system) in the direction normal through O to the plane of the wire is given by $\frac{I}{2}\left\{\frac{1}{a}\sinh^{-1}\left(\frac{a}{z}\right) - \frac{1}{\sqrt{a^2+z^2}}\right\}$.

(b) A current I flows in a helical wire of radius a which has its axis along OZ can be parameterized as $r = (a\cos\phi, a\sin\phi, \alpha\phi)$. If it has turns per unit length $\alpha = \frac{1}{2n\pi}$, show that the component of the magnetic field along the axis is nI.

4. (a) Prove that the potential at a point P of distance r due to a thin homogeneous spherical shell of matter of mass M, density σ per unit area and radius a is given by

$$\mathbb{G}(p) = \begin{cases} \frac{MG}{r} & \text{if } \ r > a \\ \\ \frac{MG}{a} & \text{if } \ r \leq a, \end{cases}$$

where G is a gravitational constant.

(b) Assume that the density of a star is a function only of the radius r measured from the center of the star and is given by $\rho = \frac{Ma^2}{2\pi r(r^2+a^2)^2}, 0 \leq r < \infty$, where M is the mass of the star, and a is a constant which determines the size of the star. Find the gravitational potential inside the star as a function of r.