

EASTERN UNIVERSITY, SRI LANKA
THIRD EXAMINATION IN SCIENCE 2005/2006

August/September' 2007

FIRST SEMESTER

MT 304 - GENERAL TOPOLOGY

Proper & Repeat

Answer all questions

Time: Two hours

Q1. (a) Define the following terms:

i. Topology on a set;

ii. Closure of a set.

[20 marks]

(b) Let X be a non-empty infinite set and let τ be the family consisting of ϕ and all subsets of X whose complements are finite. Prove that τ is a topology on X .

[30 marks]

(c) Let (X, τ) be a topological space and let $A, B, C \subseteq X$. Define $Fr(M) = \overline{M} \setminus M^\circ$ for any subset M of X . Prove that

i. If B is closed then $Fr(B) \subseteq B$.

ii. The set B is both open and closed if and only if $Fr(B) = \phi$.

iii. $\overline{A} = Fr(A) \cup A$.

[35 marks]

(d) Is the union of two topologies on a set X again a topology? Justify your answer.

[15 marks]

Q2. (a) Define the following in a topological space (X, τ) :

i. Base;

ii. Disconnected set.

[20 m]

(b) Let \mathbb{B} be a class of subsets of a non-empty set X . Prove that \mathbb{B} is a base for some topology τ on X if and only if it satisfies the following properties:

i. $X = \bigcup_{i \in I} B_i$, $B_i \in \mathbb{B}$;

ii. For any $B_\alpha, B_\beta \in \mathbb{B}$, $B_\alpha \cap B_\beta = \bigcup_{i \in I} B_i$, that is,

$B_\alpha \cap B_\beta$ is the union of members of \mathbb{B} .

[35 m]

(c) Prove that a topological space (X, τ) is disconnected if and only if there exists a non empty proper subset of X , which is both open and closed.

[20 m]

(d) Let (X, τ_X) and (Y, τ_Y) be two topological spaces and let $f : X \rightarrow Y$ be a continuous function. Prove that if $A \subseteq X$ is connected, then the image $f(A)$ is connected.

[25 m]

Q3. (a) Define the following in a topological space (X, τ) :

i. Compact set;

ii. Sequentially compact set.

[20 m]

(b) Let (X, τ) be a topological space and let (Y, τ_Y) be its subspace. Prove that $A \subseteq Y$ is compact in (Y, τ_Y) if and only if A is compact in (X, τ) .

[25 m]

(c) Prove that continuous image of a compact set is compact.

[15 m]

(d) Let A, B be two compact sets in a topological space (X, τ) . Show that $A \cup B$ is compact.

[20 m]

(e) Is $A = (0, 1)$ on the real line \mathbb{R} with usual topology compact? Justify your answer.

[20 m]

Q4. (a) Define the Frechet space and the Hausdorff space.

[15 m]

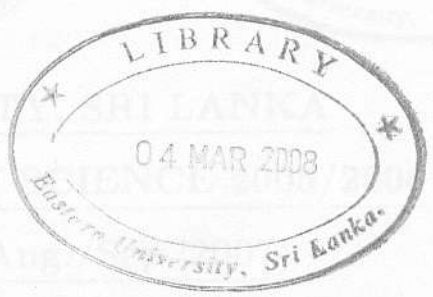
(b) Let (X, τ_1) and (Y, τ_2) be two topological spaces and let $f : X \rightarrow Y$. Show that f is continuous on $A \subseteq X$ if and only if $f(\overline{A}) \subseteq \overline{f(A)}$.

[30 m]

(c) Prove that every Hausdorff space is Frechet space. Is the converse true? Justify your answer.

[25 m]

(d) Prove that a topological space is a Frechet space if only if every singleton subset of X is closed. [30 marks]



EASTERN UNIVERSITY, SRI LANKA
EXAMINATION IN SCIENCE
FIRST SEMESTER (AUGUST 2007)
REGRESSION ANALYSIS & QUALITY CONTROL
(Proper & Repeat)

Time: Three hours

Simple linear regression? Distinguish between simple linear regression and multiple regression. State the method of least squares. Estimate the linear regression parameters by the method of least squares. Obtain the standard estimators of the above parameters. The following data were obtained at each of four depths in a river in determining the amount of dissolved oxygen varied from one depth to another. The data are as follows:

Depth (m)	Measured Oxygen (%)	Y
1	4.0, 5.0	3
2	6.0, 8.0	4
3	7.0, 8.0	5