EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE-2003/2004

(June/July'2005)

SECOND SEMESTER

REPEAT

MT 306- PROBABILITY THEORY

Answer all questions — Time : Two hours

1. (a) Let Y be a negative binomial random variable with parameters r and p and its probability mass function be given by,

$$P(Y=y)=\begin{cases}\binom{y-1}{r-1}p^r q^{y-r} & ; \quad y=r,r+1,r+2,\ldots,\\ 0 & \quad \text{otherwise}\end{cases}$$

Find

i. the expected value of Y ,

ii. the variance of Y ,

iii. the moment generating function of Y .

(b) Let X be a random variable having Gamma distribution with density function:

$$f(x)=\begin{cases}\frac{\lambda^n x^{n-1}e^{-\lambda x}}{\Gamma(n)} & ; \quad x\geq 0\\ 0 & ; \quad \text{otherwise}\end{cases}$$

where n and λ are parameters.

Find

i. the expected value of X ,

ii. the variance of X .

2. (a) State and prove the Baye's theorem.

(b) Three machines A, B and C produce, respectively, 40%, 10% and 50% of the items in a factory. The percentage of defective items produced by the machines are, respectively, 2%, 3% and 4%. An item from the factory is selected at random. If the selected item is defective, find the probability that the item was produced by machine C.

(c) Let $X_1, X_2, ..., X_n$ be independent random samples from normal population with mean μ and variance σ^2. Show that,

i. the statistic $\hat{\mu} = \frac{1}{n+1}\sum_{i=1}^{n} X_i$ is biased for μ.

ii. $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ is an unbiased estimator for σ^2.

iii. Let X and Y be independent random variables. X has the gamma distribution with parameters m and λ and Y has the gamma distribution with parameters n and λ. Show that $X+Y$ has the gamma distribution with parameters $(m+n)$ and λ.

3. (a) Determine the maximum likelihood estimators of the parameters of the following distributions:

i. Geometric population with parameter p.

ii. Exponential population with mean θ.

(b) If X is a random variable having a Binomial distribution with the parameters n and θ then show that the moment generating function of $Z = \frac{X - n\theta}{\sqrt{n\theta(1-\theta)}}$ approaches that of the standard normal distribution when $n \to \infty$.

4. (a) A random sample $X_1, X_2, ..., X_n$ is obtained from a distribution with probability density function,

$$f(x) = \frac{\beta^{\alpha} x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)} \; ; 0 \leq x < \infty,$$

where α and β are unknown parameters. Estimate α and β by using the method of moments.

(b) Show that if X is a random variable having the Poisson distribution with the parameter λ and $\lambda \to \infty$, then the moment generating function of $Z = \dfrac{X - \lambda}{\sqrt{\lambda}}$ approaches the moment generating function of the standard normal distribution.