

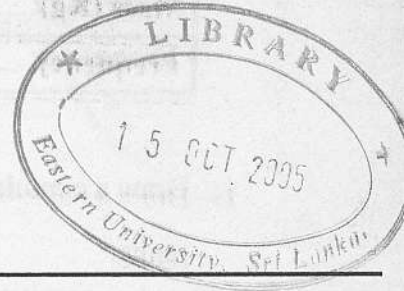
**EASTERN UNIVERSITY, SRILANKA**

**THIRD EXAMINATION IN SCIENCE 2003/2004 (Repeat)**

**SECOND SEMESTER**

**(June/July, 2005)**

**MT-308- STATISTICS**



*Answer all questions*

*Time: Two hours*

- (a) A population consists of  $n_1$  males and  $n_2$  females. The mean heights of males and females are  $\mu_1$  and  $\mu_2$  respectively and the standard deviations of the heights are  $\sigma_1$  and  $\sigma_2$  respectively. Show that the mean height of the whole population is  $\omega_1\mu_1 + \omega_2\mu_2$  and the variance is  $\omega_1\sigma_1^2 + \omega_2\sigma_2^2 + \omega_1\omega_2(\mu_1 - \mu_2)^2$

Where  $\omega_1 = \frac{n_1}{n_1 + n_2}$  and  $\omega_2 = \frac{n_2}{n_1 + n_2}$ .

Hence or otherwise, show that, if a single observation taking the value  $x$  is added to a population of size  $n$  with mean  $\mu$  and the variance  $\sigma^2$ , the new variance will be larger than the old if

$$|\mu - x| > \left(\frac{n+1}{n}\right)^{\frac{1}{2}} \sigma$$

- (b) Show that the mean deviation from the mean and the standard deviation of the arithmetic progression  $a, a+d, \dots, a+2nd$  are

$\frac{nd(n+1)}{2n+1}$  and  $d\sqrt{\frac{n(n+1)}{3}}$  respectively.

Verify that the latter is greater than the former.

2. (a) In a fishing competition, the total catches of all 44 anglers who participated in the competition, had masses (to the nearest 0.1 kg) are given in table below.

Mass (Kg)	0.3-0.7	0.8-1.2	1.3-1.7	1.8-2.2	2.3-3.7	3.8-4.2
Frequency	8	12	8	8	4	4

1. Draw a cumulative frequency curve and from it find the median and inter quartile range.
2. Calculate the mean and standard deviation of the distribution.
3. Given the additional information that all eight anglers placed in the first class inter caught 0.3 kg each, obtain the revised value for mean.

- (b) Two cricketers scored the following runs in 10 innings. Find who is a better run-getter if the consistency is the criterion for awarding a prize, who should get the prize.

<b>Cricketer A</b>	42	17	83	59	72	76	64	45	40	32
<b>Cricketer B</b>	28	70	31	0	59	108	82	14	3	95

- (c) Let  $R$  be the range and  $\sigma$  be the standard deviation of a set of observations  $x_1, x_2, \dots, x_n$ . Prove that  $R \geq \sigma$ .

[Hint:  $x_i - \mu \leq R$ ;  $i = 1, 2, \dots, n$ .]

3. (a) Show that Spearman's rank correlation coefficient  $r_s$  is given by,

$$r_s = 1 - \frac{6 \sum_{i=1}^n d_i^2}{n(n^2 - 1)}$$

where  $n$  is the number of observations and  $d_i$  is the difference between ranks assigned to the  $i^{\text{th}}$  individual.

- (b) Show that  $-1 \leq r_s \leq 1$ .

- (c) The following table gives the information on the ages ( in years ) and the number of breakdowns during the past month for a sample of 10 machines at a large company.

<b>Age</b>	2	7	3	8	13	9	4	10	11	14
<b>Number of breakdowns</b>	9	5	1	4	10	7	2	11	8	12



Find Spearman's rank correlation coefficient for the above data.

A farmer wants to find the relationship between the amount of fertilizer used and the yield of corn. He selected several acres of his land on which he used different amount of fertilizer to grow corn. The following table gives the amount of fertilizer ( in pounds) used and the yield of corn ( in bushels) for each of the seven acres.

<b>Amount of fertilizer used</b>	<b>yield of corn</b>
120	138
80	112
100	129
70	96
88	119
75	104
110	134

- (a) Draw a scatter diagram for these data. Does the scatter diagram show a linear relationship between fertilizer used and yield of corn?
- (b) Fit the estimated regression line, giving the statistical model for the data.
- (c) Give a brief interpretation of the estimated slope calculated in part (b).

- (d) Test at 5% significance level if the true slope is different from zero.
- (e) Compute the coefficient of determination and interpret it.
- (f) Find a 95% confidence interval for the true slope.
- (g) What is the estimated value of the yield of corn if the farmer used 125 pounds of fertilizer to grow?

Yield of corn (lb)	Amount of fertilizer used (lb)
110	100
115	110
120	120
125	130
130	140
135	150
140	160
145	170
150	180
155	190
160	200