EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE 2003/2004

SECOND SEMESTER (JUNE/JULY' 2005)

(Proper & Repeat)

MT 310 - FLUID MECHANICS

Answer all questions Time: Two hours

1. (a) Derive the continuity equation for an incompressible fluid flow in the form $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$ in cartesian coordinates, where u, v and w are the cartesian components of the velocity $\underline{q}$.

   (b) Show that $\frac{k}{r^5}(3x^2-r^2, 3xy, 3xz)$, where $r^2 = x^2+y^2+z^2$ and k is a constant, represents the velocity field in a possible fluid motion.

   Show also that this motion is irrotational.

   (c) Find the velocity potential and the equation of stream lines for the velocity field given in (b).

2. Let a gas occupy the region $r \leq R$, where R is a function of time t, and a liquid of constant density ρ lie outside the gas. By assuming that there is contact between the gas and the liquid all the time and that the motion is symmetric about the origin $r = 0$, show that the motion is irrotational.

   If the velocity at $r = R$, the gas liquid boundary is continuous then show that the pressure p at a point $P(\underline{r}, t)$ in the liquid is given by

   $$\frac{p}{\rho}+\frac{1}{2}\left(\frac{R^2\dot{R}}{r^2}\right)^2-\frac{1}{r}\frac{d}{dt}(R^2\dot{R})=f(t), \text{ where } r=|\underline{r}|.$$

Further, if it is given that the liquid extends to infinity and is at rest with constant pressure π at infinity, prove that the gas liquid interface pressure is equal to $\pi + \frac{1}{2}\rho R^{-2}\frac{d}{dR}(R^3\dot{R}^2)$.

If the gas obeys the Boyle's law $pv^{\frac{4}{3}}$ = constant, where v is the volume of the gas, and expands from rest at $R = a$ to a position of rest at $R = 2a$, show that the ratio of initial pressure of the gas to pressure of the liquid at infinity is 14:3.

3. (a) Let a three dimensional doublet of strength μ be situated at the origin. Show that the velocity potential ϕ at a point $P(r, \theta, \psi)$, in spherical polar coordinates, due to the doublet can be written in the form $\phi = \mu r^{-2}\cos\theta$.

(b) Three dimensional doublets of strength μ_1, μ_2 are situated at A_1 and A_2 whose cartesian coordinates are $(0, 0, c_1)$ and $(0, 0, c_2)$, their axes being directed towards and away from the origin respectively. Show that the condition for no transport of fluid across the surface of sphere $x^2 + y^2 + z^2 = c_1c_2$ is $\frac{\mu_2}{\mu_1} = \left(\frac{c_2}{c_1}\right)^{\frac{3}{2}}$.

4. Write down the Bernoulli's equation for steady motion of an incompressible inviscid fluid.

Let a fluid of density ρ fill the region of space on the positive side of the x axis with the plane determined by the y axis and z axis being a fixed boundary. If a two dimensional source of strength m is situated at the point $(a, 0)$, find the points on the boundary at which the velocity is maximum. Show that the resultant thrust on area formed by the part of the axis of y which lies between $y = \pm b$ and unit length along the z axis is

$$2p_0b - 2m^2\rho\left[\frac{1}{a}\tan^{-1}\left(\frac{b}{a}\right) - \frac{b}{a^2 + b^2}\right],$$

where p_0 is the pressure at infinity.