EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE - 2003/2004

SECOND SEMESTER

(JUNE/JULY 2005)

PH 305 FUNDAMENTALS OF STATISTICAL PHYSICS

Time: 01 hour.

Answer ALL Questions

You may find the following information useful.

Plank's constant $h = 6.625 \times 10^{-34}\ Js$

Mass of an electron $m_e = 9.109 \times 10^{-31}\ kg$

Charge of an electron $e = 1.602 \times 10^{-19}\ Coulomb$

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01. Explain the terms "partition function" and "density of states" as used in statistical physics? State the conditions for a system to obey Maxwell-Boltzmann statistics and derive an expression for the Maxwell-Boltzmann distribution function in terms of the partition function of the system.

Using Maxwell-Boltzmann distribution function, show that for a system of N molecules of an ideal gas at absolute temperature T, the number of molecules in energy range E and $E + dE$ is given by

$$N(E)dE = 2\pi N\left(\frac{1}{\pi k_B T}\right)^{\frac{3}{2}} E^{\frac{1}{2}} e^{-\frac{E}{k_B T}} dE$$ where, the symbols have their usual meanings.

Hence show that the ratio of the mean energy to the most probable energy of molecules is 3:1.

You may use the following informations useful:

The thermodynamic probability of Maxwell-Boltzmann statistics is given by $\Omega = N!\prod_{j=1}^{N}\frac{g_j^{N_j}}{N_j!}$

The density of states of the ideal gas in the energy range between E and $E + dE$ is

$$g(E)dE = 2\pi V\left(\frac{2m}{h^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}} dE$$

The partition function of the ideal gas is $Z = V\left(\frac{2\pi m k_B T}{h^2}\right)^{\frac{3}{2}}$ and

$$\int_0^\infty x^4 e^{-ax^2} dx = \frac{3}{8}\sqrt{\frac{\pi}{a^5}}$$

02. State the conditions under which a system of particles obeys Fermi-Dirac statistics. Identifying clearly the quantities involved, write down the expression for Fermi-Dirac distribution law.

(a) Use the above expression for free electrons in a metal to show that at $T = 0$, the Fermi energy is given by

$$E_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{\frac{2}{3}}$$, where the symbols have their usual meanings.

Calculate the Fermi energy in Copper using the following data:

Density of Copper $= 8.94\times10^3\, kgm^{-3}$

Atomic mass of Copper $= 63.5 a.m.u$ and

$1 a.m.u = 1.66\times10^{-27}\, kg$

(b) Show that the mean energy of a free electron is given by $\frac{3}{5}E_F$. Hence briefly discuss the significance of E_F.

You may use the degeneracy function $g(E)$ for free electrons in a metal given by

$$g(E)dE = 4\pi V\left(\frac{2m}{h^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}} dE$$