EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE (2004/2005)

FIRST SEMESTER (Jan./ Feb., 2006)

MT 305 - OPERATIONAL RESEARCH

Answer all questions **Time allowed : Three hours**

1. Define the "feasible region for a liner programming problem".

A firm manufactures two products A and B which are sold at 8 cents and 15 cents per unit respectively, the market for both products being virtually unlimited. A is processed on machine I and B is processed on machine II. Then both are packaged at the packing plant. Raw material costs 6 cents per kg and is processed at 5000 kg per hour by machine I with 10% wastage; it is processed at 4000 kg per hour by machine II and 20% is wasted. Machine I is available 6 hours a day with a running cost of \$288 per hour; machine II is available 5 hours a day with a running cost of \$336 per hour. Finished units of A weight $\frac{1}{4}$ *kg* and units of B weight $\frac{1}{3}$ *kg*. The packing plant is available 10 hours a day at a cost of \$360 per hour. Units of A can be packed at 12000 per hour whilst units of B are packed at the slower rate of 8000 units per hour.

The company wishes to determine those values for x_1 and x_2, the input (in thousands of kg) of raw material used for the products A and B respectively, so as to maximize daily profit.

Formulate this problem as a linear programming problem and calculate the optimal solution graphically.

(You may assume 100 cents equal to 1\$)

2. A florist is planning to make up floral arrangements for the upcoming festive season. The florist has four varieties of flower as specified in the following table.

Variety	Cost per flower(Rs.)	Availability
White Roses	4	900
Red Roses	6	800
Yellow Roses	5	450
Carnations	4	1000

The flowers can be used in any of the three popular arrangements; Duluxe Rose, Spring Color and Economy. The make up and the selling price for each arrangement are shown below.

Arrangement	Composition	Selling Price(Rs.)
Duluxe Rose	15 white Roses 15 red Roses 10 yellow Roses	300
Spring Color	10 white Roses 10 red Roses 6 yellow Roses 10 carnations	250
Economy	5 red Roses 2 yellow Roses 10 carnations	150

The florist wishes to determine how many units of each arrangements should be made up in order to maximize the total profit.

(a) Formulate the above problem as a linear programming problem.

(b) Use Simplex method to find the optimum solution.

3. Use the Revised Simplex method to

minimize $z = -9x_1 - 10x_2 - 15x_3$

subject to

$$x_1 + 2x_2 + 5x_3 \leq 36$$

$$2x_1 + 3x_2 + 3x_3 \leq 48$$

$$x_1 + x_2 + 2x_3 \leq 22$$

$$x_i \geq 0, \ i = 1, 2, 3.$$

The following four identities may help you in your computation.

$$\begin{pmatrix} 5 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}^{-1} = \frac{1}{5}\begin{pmatrix} 1 & 0 & 0 \\ -3 & 5 & 0 \\ -2 & 0 & 5 \end{pmatrix}, \quad \begin{pmatrix} 5 & 0 & 1 \\ 3 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 0 & -1 \\ 1 & 3 & -7 \\ -2 & 0 & 5 \end{pmatrix},$$

$$\begin{pmatrix} 5 & 2 & 1 \\ 3 & 3 & 2 \\ 2 & 1 & 1 \end{pmatrix}^{-1} = \frac{1}{4}\begin{pmatrix} 1 & -1 & 1 \\ 1 & 3 & -7 \\ -3 & -1 & 9 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 2 \\ 0 & 1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & -1 & 3 \end{pmatrix}.$$

4. Briefly explain the Vogel's approximation method.

A company has factories at A, B and C which supply warehouses at D, E, F and G. The factory capacities are 230, 280 and 180 respectively for regular production. If overtime production is utilized, the capacities can be increased to 300, 360 and 190 respectively. The current warehouse requirements are 165, 175, 205 and 165 respectively. Unit shipping costs in rupees between the factories and the warehouses are given as follows:

		Warehouses			
		D	E	F	G
Factories	A	7	8	9	11
	B	5	11	8	7
	C	4	23	3	12

Determine the optimum distribution for the company to minimize costs if the increment unit over costs are Rs.5, Rs.4 and Rs.6 respectively.

5. Give a brief explanation for Mack's method to find the solution of an Assignment method.

On a particular day a haulage company has to pick up 5 loads at the points A,B,C,D,E and deliver them to the points a,b,c,d,e. The distances (in miles) between the pick up points and final destinations of the loads are shown in the table below:

$A \to a$	$B \to b$	$C \to c$	$D \to d$	$E \to e$
60	30	100	50	40

The firm has available 5 lorries of two types X and Y at the points S,T,U,V,W the type of lorry being- type X at S, type Y at T, type X at U, type X at V and type Y at W. The type X lorries are newer and more versatile than type Y and have lower costs associated with them. The total running costs (Rupees) per mile of two types of lorry are shown below.

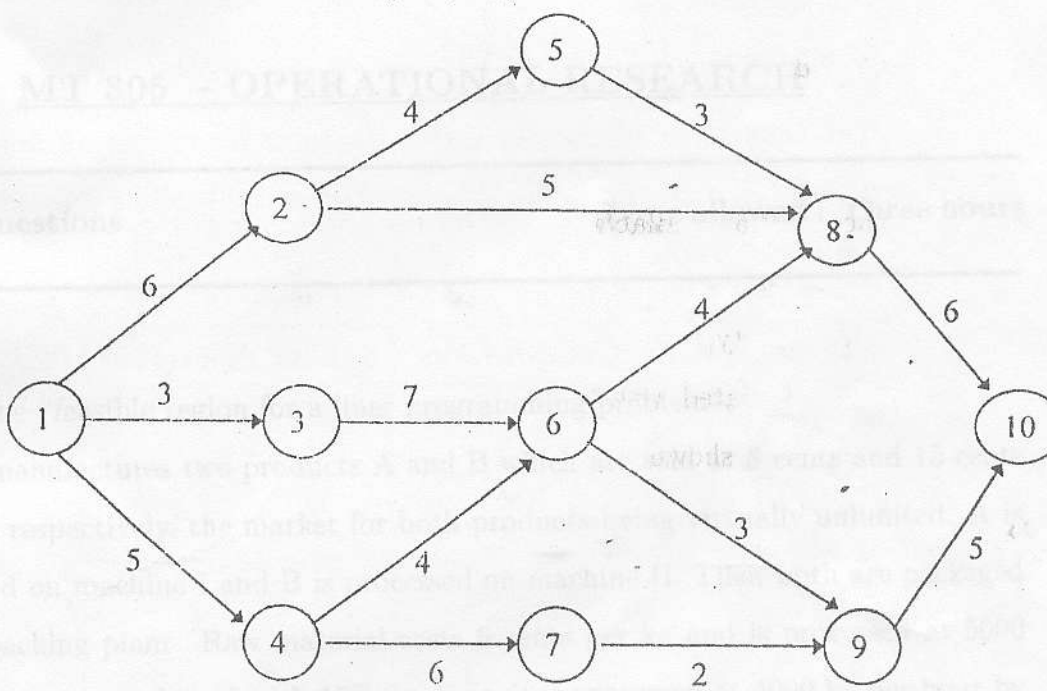
Type	**Empty**	**Loaded**
X	20	40
Y	30	60

The distances (in miles) of the initial positions of the lorries from the loading points are shown below.

	A	B	C	D	E
S	30	20	40	10	20
T	30	10	30	20	30
U	40	10	10	40	10
V	20	20	40	20	30
W	30	20	10	30	40

Use Hungarian method to show that, there are more than one possible allocations for this problem. Determine the allocations of the lorries to the loads which minimizes the costs. It should be assumed that all the loads are of about the same size and will require the same amount of packing, handling etc.

6. Find the maximum flow for the following network.



by

- Intuitive technique,
- Labelling technique.