# EASTERN UNIVERSITY, SRI LANKA

## THIRD EXAMINATION IN SCIENCE 2003/2004

## SECOND SEMESTER (MARCH / APRIL' 2006)

## (Repeat)

## MT 310 - FLUID MECHANICS

Answer all questions | Time allowed: Two hours

1. (a) Derive the equation for equilibrium, $\underline{F} = \frac{1}{\rho}\nabla p$, where $\underline{F}$, $p$ and $\rho$ are body force per unit mass, static fluid pressure and the density of the fluid respectively.

   (b) Let the density and pressure in a self gravitating gas be connected by the relation $p = C^2\rho$, where $C$ is a constant.

   Prove that if the gas is in equilibrium then $C^2\nabla^2 \ln\rho + 4\pi G\rho = 0$, where $G$ is the gravitational constant.

   Further show that if $\rho$ depends on a single linear coordinate $x$ and the gas is unbounded then $\rho = \rho_o \operatorname{sech}^2\left[\frac{x}{C}\sqrt{2\pi G\rho_o}\right]$ if $\frac{d\rho}{dx} = 0$ and $\rho = \rho_o$ when $x = 0$.

2. (a) Let a gas occupy the region $r \leq R$, where $R$ is a function of time $t$, and a liquid of constant density $\rho$ lie outside the gas. By assuming that there is contact between the gas and the liquid all the time and that the motion is symmetric about the origin $r = 0$, show that the motion is irrotational.

   If the velocity at $r = R$, the gas liquid boundary is continuous then show that the pressure $p$ at a point $P(\underline{r}, t)$ in the liquid is given by

   $$\frac{p}{\rho} + \frac{1}{2}\left(\frac{R^2\dot{R}}{r^2}\right)^2 - \frac{1}{r}\frac{d}{dt}(R^2\dot{R}) = f(t), \text{ where } r = |\,\underline{r}\,|.$$

(b) A sphere whose center is fixed and whose radius at time t is $a + b\cos\omega t$, where $a, b$ and $\omega$ being positive constants, is surrounded by an infinite mass of liquid of uniform density $\rho$, on which no body forces act. If the pressure at infinity is $\pi$, prove that the pressure at the surface of the sphere is $\pi + \frac{1}{4}b\rho\omega^2\,[b - 5b\cos 2\omega t - 4a\cos\omega t]$. Find the least value of this pressure.

3. (a) Let a two dimensional source of strength $m$ is situated at origin. Show that the complex potential $w$ at a point $\mathrm{P}(z)$ due to this source is given by $w = -m\ln z$.

(b) Two sources, each of strength $m$ placed at $(-a, 0), (a, 0)$ and a sink of strength $2m$ at the origin. Show that the streamlines are the curves

$$(x^2 + y^2)^2 = a^2(x^2 - y^2 + \lambda xy),$$

where $\lambda$ is a variable parameter. Show also that the fluid velocity at any point is $\dfrac{2ma^2}{(r_1 r_2 r_3)}$, where $r_1, r_2$ and $r_3$ are the distances of points from the sources and the sink.

4. Write down the Bernoulli's equation for steady motion of an incompressible inviscid fluid. If fluid fills the region of space on the positive side of the $x$ axis, which is a rigid boundary and if there is a source $m$ at the point $(0, a)$ and an equal sink at $(0, b)$ and if the pressure on the negative side is the same as the pressure at infinity, show that the resultant pressure on the boundary is $\dfrac{\pi\rho m^2(a-b)^2}{2ab(a+b)}$, where $\rho$ is the density of the fluid and $a > b$.