EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE (2004/2005)

FIRST SEMESTER (Jan./Feb., 2006)

ST 303 - REGRESSION ANALYSIS & QUALITY CONTROL

Answer all questions **Time allowed: Three hours**

1. A group of scientists collected a number of specimens of a substance from various locations. All collecting points, the mean temperature(X) and the average magnesium content (Y) of the specimens are noted. The results are given below.

X	18.1	23	17.5	20.2	14.7	13.8	15.1	13.9	24.2
Y	8.8	9.5	8.9	9.1	8.6	8.3	8.5	8.2	9.5

(i) Find the linear regression of magnesium content on the mean temperature.

(ii) Construct the analysis of variance and test the hypothesis that the slop of the regression line is zero.

(iii) Give a 95% confidence interval for the expected value of Y at $X = 25$.

2. The data below gives a number of measurements of water flow (x) and salinity (y).

Water flow(x)	23	24	26	25	30	24	23	22	29	24	25	28	22	22	24
Salinity(y)	7.6	7.7	5.4	5.9	5.0	6.5	8.3	8.2	5.2	8.2	6.0	4.9	8.7	8.1	6.0

$S_{xx} = 88.93,\quad S_{xy} = -42.18,\quad S_{yy} = 26.46.$

(i) Fit a relationship of the form $y = \hat{\alpha} + \hat{\beta}\, x$.

(ii) Compute the analysis of variance table.

(iii) Partition the residual sum of squares into pure error and lack of fit and comment on the adequacy of the fit of your straight line.

(iv) Using the fitted relationship estimate the mean salinity when the water flow is 25 and give the standard error of your estimate.

3. (a) A general linear model relating a response variable y to $p-1$ predictor variables $X_1, X_2, \cdots, X_{p-1}$ is given by

$$E(y) = X\beta, \qquad \text{Var}(y) = I\sigma^2$$

where X is the $1 \times p$ design matrix, β is a $p \times 1$ vector of parameters and I is the identity matrix of order p.

Drive the least squares estimator $\hat{\beta}$ of β and obtain its expectation and variance-covariance matrix.

(b) In an experiment, a scientist has made four observations 3, 30, 20, 18, each assumed to have the same variance σ^2. Their expected values can be expressed in terms of two unknown quantities, u and v, as follows:

$$\begin{aligned} 2u - v &= 3 \\ u + 3v &= 30 \\ u + v &= 20 \\ 3u + v &= 18 \end{aligned}$$

Find

(i) the least squares estimates of u and v.

(ii) an estimate of σ^2.

(iii) covariance matrix of u and v.

4. A company uses statistical quality control for its products. 20 samples of 5 items were selected to set up quality control charts. The value of $\overline{X}$ and R for each sample are as follows:

$\overline{X}$	R	$\overline{X}$	R	$\overline{X}$	R	$\overline{X}$	R
34.0	4	32.2	2	35.8	4	31.6	5
31.6	4	33.0	5	38.4	4	33.0	5
30.8	2	32.6	13	34.0	14	28.2	3
33.0	3	33.8	19	35.0	4	31.8	9
35.0	5	37.8	6	33.8	7	35.6	6

Draw the $\overline{X}$ and R charts showing the upper and lower control limits. Plot the 20 values and state whether the system is under statistical control.

5. In a watch glass producing factory, a random sample of 50 items are taken from each days output. Assume that the production of glasses is a continuous process. Results of 15 days past operations are as follows.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of defectives	4	9	10	11	13	30	26	13	8	23	34	25	18	12	4

Construct a P chart for future use.

6. (a) The sampling inspection scheme states that from each batch (large) take a random sample of 20 pieces. If the sample contains less than 2 defectives accept the batch, otherwise reject the batch. Evaluate the probability of acceptance for $p' = 1\%,\ 2\%,\ 4\%,\ 7\%,\ 10\%,\ 15\%,\ 20\%$ and plot the OC curve.

   (b) A double sampling plan calls for a first sample of 50 items to be inspected. If no defective are found the batch is accepted. If more than two defective are found the batch is rejected. Otherwise a second sample of 100 is drawn and the batch is accepted if the combined samples contain less than 4 defectives. If the process average $p = 0.01$, what is the probability that the batch will be

      i. accepted on the first sample.

      ii. rejected on the second sample.

      iii. finally accepted.