

**EASTERN UNIVERSITY, SRI LANKA**  
**DEPARTMENT OF MATHEMATICS**  
**SECOND EXAMINATION IN SCIENCE -2008/2009**  
**SECOND SEMESTER (Sept./Oct., 2010)**  
**MT 218 - FIELD THEORY**  
**(PROPER & REPEAT)**

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Answer all Questions

Time: Two hours

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1. State the *Coulomb's law* in an electric field.
  - (a) Define the term *electric field strength* due to a point charge.
    - i. A uniformly charged disk of radius  $R$  with a total charge  $Q$  lies in the  $xy$ -plane. Find the electric field at a point  $P$ , along the  $z$ -axis that passes through the center of the disk perpendicular to its plane. Discuss the limit where  $R \gg z$ .
    - ii. Two infinite plane sheets are separated by a distance ' $d$ '. The first has a charge density  $+\sigma$  and the second has a charge density  $-\sigma$ . Find the electric field intensity at any point between them.
  - (b) A thin rod extends along the  $z$ -axis from  $z = -d$  to  $z = d$ . The rod carries a positive charge  $Q$  uniformly distributed along its length  $2d$  with charge density  $\lambda = \frac{Q}{2d}$ .
    - i. Calculate the electric potential at a point  $z > d$  along the  $z$ -axis.
    - ii. What is the change in potential energy if an electron moves from  $z = 4d$  to  $z = 3d$ ?
    - iii. If the electron started out at rest at the point  $z = 4d$ , what is its velocity at  $z = 3d$ ?

2. State the *Gauss's theorem* in an electric field.

(a) Define the term *electric flux*.

i. Show that the electric flux through a square surface of edges  $2l$  due to a charge  $+Q$  located at a perpendicular distance  $l$  from the center of the square is  $\frac{Q}{6\epsilon_0}$ , where  $\epsilon_0$  is the permittivity constant.

ii. Using the result obtained in the above part, if the charge  $+Q$  is now at the center of a cube of side  $2l$ , find the total flux emerging from all the six faces of the closed surface.

(b) Define the term *electric dipole*.

Prove that the electric potential  $V$  at a point  $Q$  at a distance  $r$  from the dipole of moment  $\underline{P}$  is given by

$$V = -\frac{1}{4\pi\epsilon_0} \left\{ \underline{P} \cdot \text{grad} \left( \frac{1}{r} \right) \right\}$$

and the electric field due to the dipole is given by

$$\underline{E} = \frac{1}{4\pi\epsilon_0} \left\{ \frac{3(\underline{P} \cdot \underline{r})\underline{r}}{r^5} - \frac{\underline{P}}{r^3} \right\}$$

3. (a) Using the separation of variables or otherwise, show that the appropriate separable solution of the Laplace equation  $\nabla^2\phi = 0$ , where  $\phi$  is a potential function in three dimensional rectangular coordinates is given by

$$\phi(x, y, z) = (Ae^{\sqrt{(k^2+l^2)x}} + Be^{-\sqrt{(k^2+l^2)x}})(C \sin ky + D \cos ky)(E \sin lz + F \cos lz),$$

where  $A, B, C, D, E, F, k$  and  $l$  are arbitrary constants.

(b) An infinitely long rectangular metal pipe (side  $a$  and  $b$ ) is grounded, but one end, at  $x = 0$ , is maintained at a specified potential  $\phi_0(y, z)$ . Show that the potential inside the pipe subject to the boundary conditions:

i.  $\phi = 0$  when  $y = 0$ ;

ii.  $\phi = 0$  when  $y = a$ ;

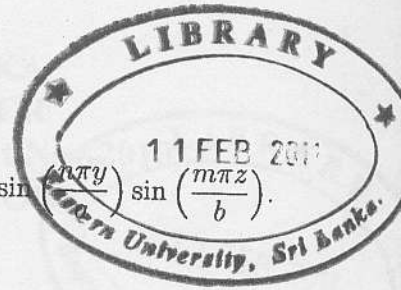
iii.  $\phi = 0$  when  $z = 0$ ;

iv.  $\phi = 0$  when  $z = b$ ;

v.  $\phi \rightarrow 0$  as  $x \rightarrow \infty$ ;

vi.  $\phi = \phi_0(y, z)$ , when  $x = 0$ ; is given by

$$\phi(x, y, z) = \frac{16\phi_0}{\pi^2} \sum_{n,m=1,3,5,\dots} \frac{1}{nm} e^{-\pi\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2} x} \sin\left(\frac{n\pi y}{a}\right) \sin\left(\frac{m\pi z}{b}\right).$$



4. (a) Define the *magnetic flux density*  $\underline{B}$  and show that  $\text{div } \underline{B} = 0$  in space.

State the *Ampere's law* in integral form and deduce that

$\text{Curl } \underline{B} = \mu_0 \underline{J}$ , where  $\underline{J}$  is the current density.

(b) State the *Biot - Savart law*.

Find the magnetic field at a distance  $d$  from an infinitely long wire which flow a current  $I$ .

Hence calculate the magnetic field at the center of a current carrying square coil of a wire with sides  $2a$ .

(c) Consider a closed semi circular loop lying in the  $xy$  plane carrying a current  $I$  in the counter clockwise direction. If a uniform magnetic field is applied in the positive  $y$  direction, find the magnetic force acting on the straight segment and the semi circular portion.