EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
SECOND EXAMINATION IN SCIENCE -2008/2009
SECOND SEMESTER (Sept./Oct., 2010)
MT 218 - FIELD THEORY
(PROPER & REPEAT)

Answer all Questions **Time: Two hours**

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1. State the *Coulomb's law* in an electric field.

   (a) Define the term *electric field strength* due to a point charge.

   i. A uniformly charged disk of radius R with a total charge Q lies in the xy-plane. Find the electric field at a point P, along the z-axis that passes through the center of the disk perpendicular to its plane. Discuss the limit where $R \gg z$.

   ii. Two infinite plane sheets are separated by a distance 'd'. The first has a charge density $+\sigma$ and the second has a charge density $-\sigma$. Find the electric field intensity at any point between them.

   (b) A thin rod extends along the $z-$axis from $z = -d$ to $z = d$. The rod carries a positive charge Q uniformly distributed along its length $2d$ with charge density $\lambda = \dfrac{Q}{2d}$.

   i. Calculate the electric potential at a point $z > d$ along the $z-$axis.

   ii. What is the change in potential energy if an electron moves from $z = 4d$ to $z = 3d$?

   iii. If the electron started out at rest at the point $z = 4d$, what is its velocity at $z = 3d$?

2. State the *Gauss's theorem* in an electric field.

   (a) Define the term *electric flux*.

      i. Show that the electric flux through a square surface of edges $2l$ due to a charge $+Q$ located at a perpendicular distance l from the center of the square is $\frac{Q}{6\epsilon_0}$, where ϵ_0 is the permiability constant.

      ii. Using the result obtained in the above part, if the charge $+Q$ is now at the center of a cube of side $2l$, find the total flux emerging from all the six faces of the closed surface.

   (b) Define the term *electric dipole*.

   Prove that the electric potential V at a point Q at a distance r form the dipole of moment $\underline{P}$ is given by

$$V = -\frac{1}{4\pi\varepsilon_0}\left\{\underline{P} \cdot grad\left(\frac{1}{r}\right)\right\}$$

   and the electric field due to the dipole is given by

$$\underline{E} = \frac{1}{4\pi\varepsilon_0}\left\{\frac{3(\underline{P}\cdot\underline{r})\underline{r}}{r^5} - \frac{\underline{P}}{r^3}\right\}.$$

3. (a) Using the separation of variables or otherwise, show that the appropriate separable solution of the Laplace equation $\nabla^2\phi = 0$, where ϕ is a potential function in three dimensional rectangular coordinates is given by

$$\phi(x,y,z) = (Ae^{\sqrt{(k^2+l^2)}\,x} + Be^{-\sqrt{(k^2+l^2)}\,x})(C\sin ky + D\cos ky)(E\sin lz + F\cos lz),$$

   where A, B, C, D, E, F, k and l are arbitrary constants.

   (b) An infinitely long rectangular metal pipe (side a and b) is grounded, but one end, at $x = 0$, is maintained at a specified potential $\phi_0(y,z)$. Show that the potential inside the pipe subject to the boundary conditions:

      i. $\phi = 0$ when $y = 0$;

      ii. $\phi = 0$ when $y = a$;

      iii. $\phi = 0$ when $z = 0$;

      iv. $\phi = 0$ when $z = b$;

v. $\phi \to 0$ as $x \to \infty$;

vi. $\phi = \phi_0(y, z)$, when $x = 0$; is given by

$$\phi(x, y, z) = \frac{16\phi_0}{\pi^2} \sum_{n,m=1,3,5,\ldots} \frac{1}{nm} e^{-\pi\sqrt{(\frac{n}{a})^2+(\frac{m}{b})^2}\, x} \sin\left(\frac{n\pi y}{a}\right) \sin\left(\frac{m\pi z}{b}\right).$$

4. (a) Define the *magnetic flux density* $\underline{B}$ and show that $div\ \underline{B} = 0$ in space.
State the *Ampere's law* in integral form and deduce that
$Curl\ \underline{B} = \mu_0\ \underline{J}$, where $\underline{J}$ is the current density.

(b) State the *Biot – Savart law.*
Find the magnetic field at a distance d from an infinitely long wire which flow a current I.
Hence calculate the magnetic field at the center of a current carrying square coil of a wire with sides $2a$.

(c) Consider a closed semi circular loop lying in the xy plane carrying a current I in the counter clockwise direction. If a uniform magnetic field is applied in the positive y direction, find the magnetic force acting on the straight segment and the semi circular portion.