EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE - 2004/2005

FIRST SEMESTER (Nov./Dec., 2006)

MT 304 - GENERAL TOPOLOGY

Special Repeat

Answer all questions **Time allowed : Two hours**

---

1. (a) Define the following terms:

   - Topology on a set,
   - Subspace of a topology,
   - Base for a topology.

   Let X be a non-empty set and let τ be the family consisting of the empty set Φ and all those non-empty subsets of X, whose complements are finite. Show that (X, τ) is a topological space.

   (b) Let (Y, τ_Y) be a subspace of a topological space (X, τ). Prove that $A \subseteq Y$ is closed in (Y, τ_Y) if and only if $A = F \cap Y$ for some closed subset F of X in (X, τ).

   (c) Let $\mathbf{B}$ be a base for a topology τ on X and let $S \subseteq X$. Prove that the collection $B_S = \{U \cap S : U \in \mathbf{B}\}$ is a base for the subspace topology τ_S on S.

2. Let f be a function from a topological space (X, τ_X) to a topological space (Y, τ_Y). Prove the following:

   (a) f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y.

   (b) f is continuous if and only if $f^{-1}(H)$ is closed in X whenever H is closed in Y.

   (c) f is continuous if and only if $f^{-1}(B^\circ) \subseteq (f^{-1}(B))^\circ, \quad \forall\, B \subseteq Y$.

3. Explain what is meant by the statement that A is a compact subset of a topological space (X, τ).

Prove the following:

(a) If A is a closed subset of a compact space (X, τ) then A is compact,

(b) Continuous image of a compact subset in a topological space is compact,

(c) If $\{A_\alpha : \alpha \in I$ and A_α are closed subsets in $X\}$ satisfies the finite intersection property then $\bigcap_{\alpha \in I} \neq \Phi$ if and only if (X, τ) is compact, where I is an index set.

4. Define the **Frechet space** (T_1) and the **Hausdorff space** (T_2).

(a) Prove that every Hausdorff space is Frechet space.

Is the converse true? Justify your answer.

(b) If A be a non-empty proper compact subset of a Hausdorff space then prove that A is closed.

(c) If A and B are two non-empty disjoint proper compact subsets of a Hausdorff space then show that there exist two disjoint open subsets G and H such that $A \subseteq G$ and $B \subseteq H$.