EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE (2004/2005)

SECOND SEMESTER (Oct./Nov.'2006)

MT 307 - CLASSICAL MECHANICS III

Answer all Questions **Time: 03 hours**

1. Tow frames of reference S and S' have a common origin O and S' rotates with an angular velocity Ω relative to S. If a moving particle P has its position vector as $\underline{r}$ relative to O at time t, prove that the acceleration of P with respect to S is

$$\frac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\Omega} \wedge \frac{\partial \underline{r}}{\partial t} + \frac{\partial \underline{\Omega}}{\partial t} \wedge \underline{r} + \underline{\Omega} \wedge (\underline{\Omega} \wedge \underline{r}),$$

A projectile is fired upwards from a point on the surface of the earth at a latitude λ, with a velocity of magnitude v_0. Assuming that the angular velocity of the earth about its axis is a constant Ω, prove that after time t the projectile is deflected east of the vertical by the amount

$$\frac{1}{3}\Omega g t^3 \cos\lambda - \Omega t^2 v_0 \cos\lambda.$$

If h is the maximum height that the projectile can reach then show that it hits the earth at a point on the horizontal plane through O, at a distance

$$\frac{4}{3}\Omega \cos\lambda \sqrt{\frac{8h^3}{g}}$$

to the west of the point of projection.

2. (a) With the usual notation, obtain the equations:

i. $\dfrac{d\underline{H}}{dt} = \sum_{i=1}^{N} \underline{r}_i \wedge \underline{F}_i$,

ii. $\underline{H} = \underline{r}_G \wedge M\underline{v}_G + \underline{H}_G$,

iii. $\dfrac{d\underline{H}_G}{dt} = \sum_{i=1}^{N} \underline{R}_i \wedge \underline{F}_i$, for a system of N particles moving in space.

(b) A pendulum consisting of a uniform rigid rod of length L and mass M, is pivoted at one end O, and swings in a vertical plane. Initially a bug of mass $\dfrac{M}{3}$ is at O and the pendulum makes an angle θ_0, ($\theta_0 << 1$ rad) with the downward vertical. As soon as the pendulum is released from rest the bug starts to crawl slowly with constant speed V along the rod towards the bottom end of the pendulum. Find the frequency ω of the oscillation of the pendulum when the bug has crawled a distance l along the rod.

3. Establish Euler's dynamical equations of motion for the motion of a rigid body with one point fixed.

A body moves about a point O under no forces. The principle moments of inertia at O are $3A$, $5A$ and $6A$ and initially the angular velocity has components $\omega_1 = n$, $\omega_2 = 0$, $\omega_3 = n$ respecting about the principle axes. Show that at any time t,

$$\omega_2 = \frac{3n}{\sqrt{5}} \tanh\left(\frac{nt}{\sqrt{5}}\right),$$

and that the body ultimately rotates about the mean axis.

4. Obtain the Lagrange's equations from the D'Alembert principle for a holonomic system.

A bead of mass m slides freely on a frictionless circular wire of radius b and mass M that rotates in a horizontal plane about a point on the circular wire with constant angular velocity ω.

(a) Obtain Lagrange's equations of motion of the system.

(b) Show that the bead oscillates as a pendulum of length $l = \dfrac{g}{\omega^2}$.

5. (a) Define the Hamiltonian H in terms of the Lagrangian L.

Show, with usual notation, that the Hamiltonian equations are given by

$$\dot{q}_j = \frac{\partial H}{\partial p_j}, \quad \dot{p}_j = -\frac{\partial H}{\partial q_j} \text{ and } \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}.$$

Show that if the Lagrangian is not an explicit function of time and the system is conservative, then the Hamiltonian H is a constant and equal to the total kinetic energy of the system.

(b) A particle of mass m at a point P is attracted to a fixed point O by an inverse square force $F_r = \frac{-GM}{r^2} = -\frac{k}{r^2}$, where $OP = r$. Find the Hamiltonian equations of motion.

6. Define the Poisson Bracket.

(a) With usual notations prove that

i. $[f, f] = 0$

ii. $[f, c] = 0$, where c is a constant.

iii. $[fg, h] = [f, h]g + f[g, h]$.

(b) Show that the Hamiltonian equations of a holonomic system may be written in the form

$$\frac{\partial H}{\partial p_k} = -[H, q_k], \quad \frac{\partial H}{\partial q_k} = [H, p_k],$$

and show that for any function $F(p_i, q_i, t)$

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + [F, H].$$

Prove that if F and G are constants of motion then $[F, G]$ is also a constant of motion.