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# EASTERN UNIVERSITY, SRI LANKA

## THIRD EXAMINATION IN SCIENCE - 2004/2005

## SECOND SEMESTER (Oct./ Nov., 2006)

## MT 310 - FLUID MECHANICS

### Proper and Repeat

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Answer all questions **Time allowed : Two hours**

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1. (a) Write the Euler's equation of motion for a fluid.

   Deduce, with usual notation, that the Bernoulli's equation is given by $\int \frac{dp}{\rho} + \frac{1}{2}q^2 + \Omega =$ constant, for irrotational motion.

   (b) A long pipe is of length $l$ and has slowly tapering cross-section. It is inclined at an angle $\alpha$ to the horizontal and water flows steadily through it from the upper end to the lower end. The section at the upper end has twice the radius of the lower end. At the lower end the water is delivered at atmospheric pressure $\pi$. If the pressure at the upper end is twice atmospheric. Show that the velocity of the delivery is $\left\{\frac{32}{15}(gl\sin\alpha + \frac{\pi}{\rho})\right\}^{\frac{1}{2}}$, where $\rho$ is the density of the water.

2. Infinite inviscid fluid of constant density is attracted towards a fixed point $O$ by a force $f(r)$ per unit mass, where r is the distance of any point from $O$. Initially the liquid is at rest, and there is a cavity bounded by a sphere of radius $a$. If the pressure at infinity and in the cavity are equal, prove that the radius $R$ of the cavity at time $t$ is such that $3\dot{R}^2 + 2R\ddot{R} + 2\int_R^\infty f(r)dr = 0$. If $f(r) = kr^{-\frac{3}{2}}$, where $k$ is a constant, then show that the cavity will be filled up after an interval time $\sqrt{\frac{2}{5k}}\, a^{\frac{5}{4}}$.

3. In the part of an infinite plane bounded by a circular quadrant $AB$ and the radii $OA$, $OB$, there is a two-dimensional motion due to a source of strength $m$ at $A$ and a sink of strength $m$ at $B$. Find the velocity potential of the motion and show that the fluid which issues from $A$ in the direction making an angle $\alpha$ with $OA$ follows the path whose polar equation is

$$r = a\sin^{\frac{1}{2}} 2\theta \left[\cot\alpha + \sqrt{\cot^2\alpha + \operatorname{cosec}^2 2\theta}\right]^{\frac{1}{2}}.$$

4. (a) Let a three dimensional doublet of strength $\mu$ be situated at the origin. Show that the velocity potential $\phi$ at a point $P(r, \theta, \psi)$, in spherical polar coordinates, due to the doublet can be written in the form $\phi = \mu r^{-2}\cos\theta$.

(b) A three dimensional doublet of strength $\mu$ whose axis is in the direction $\overrightarrow{OX}$ is distant $a$ from the rigid plane $x = 0$ which is the sole boundary of liquid of constant density $\rho$, infinite in extent. If the pressure at infinity is $\pi$, find the pressure at a point on the boundary distant $r$ from $O$. Show that the pressure on the plane is least at a distance $a\dfrac{\sqrt{5}}{2}$ from the doublet.