EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

THIRD EXAMINATION IN SCIENCE -2008/2009

FIRST SEMESTER (Sep./Oct., 2010)

MT 306 - PROBABILITY THEORY

(PROPER & REPEAT)

Answer all Questions **Time: Two hours**

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1. (a) State the Baye's theorem.

An automobile insurance company classifies drivers as class A (good risks), class B (medium risks), and class C (poor risks). They believe that class A risks constitute 30% of the drivers who apply them for insurance, class B 50% and class C 20%. The probability for a class A driver who applied for insurance, meet an accident within 12 month period is 0.01, for class B driver it is 0.1. The insurance company sells Mr.Janes an insurance policy and he met an accident within 12 months. What is the probability for that he is a class A driver? Assume that all the given events are mutually exclusive and exhaustive.

(b) Define the term conditional probability.

Weather records indicates that the probability that a particular day is dry is 3/10. SSC is a football team whose record of success is better on dry days than on wet days. The probability that SSC win on a dry day is 3/8, whereas the probability that they win on a wet day is 3/11. SSC are due to play their next match on saturday.

i. What is the probability that SSC will win.

ii. Three saturday ago SSC won their match. What is the probability that it's was a dry day.

2. (a) From a group of three Republicans, two Democrats and one Independent,a committee of two people is to be randomly selected. Let Y_1 denote the number of Republicans and Y_2 the number of Democrats on the committee.

   i. Find the joint probability distribution of Y_1 and Y_2, and then find the marginal distribution of Y_1.

   ii. Is Y_1 independent of Y_2?

   iii. Given that one of the two people on the committee is a democrat, find the conditional distribution for the number of republicans selected for the committee.

   (b) Suppose that a unit of mineral ore contains a proportion Y_1 of metal A and a proportion Y_2 of metal B. Experience has shown that the joint probability density function of (y_1, y_2) is uniform over the region $0 \leq y_1 \leq 1,\ 0 \leq y_2 \leq 1$ and $0 \leq y_1 + y_2 \leq 1$. Let $U = Y_1 + Y_2$, the proportion of metals A and B per unit.

   i. Find the probability density function for U.

   ii. Find $E(U)$ by using the part(i).

3. (a) A certain type of elevator has a maximum weight capacity Y_1,which is normally distributed with a mean of 5000 pounds and a standard deviation of 300 pounds. For a certain building equipped with this type of elevator, the elevator loading Y_2 is normally distributed random variable with a mean of 4000 pounds and a standard deviation of 400 pounds. For any given time that the elevator is in use, find the probability that it will be overloaded, assuming that Y_1 and Y_2 are independent.

(b) The manufacturer of colour television sets offers one year warranty of free replacement if the picture tube fails. He estimates the time to failure is T in years to be a random variable with the following probability density function

$$f(t) = \frac{1}{4}e^{-\frac{1}{4}t} \quad t > 0.$$

i. What percentage of the sets will we have to service?
(Hint: Assume $e^{-\frac{1}{4}} = 0.7788$)

ii. If the profit per sale is $Rs.2000$ and the replacement of picture tube cost $Rs.200$, find the expected profit per sale.

4. (a) Define the term unbiased estimator.

Given that $X_i \sim N(\mu, \sigma^2)$ for $i = 1 \ldots, n$. Then show that $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is an unbiased estimator of μ and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ is an unbiased estimator of σ^2.

(b) A Small amount of the trace element selenium, from 50 to 200 micrograms(μg) per day is considered essential to good health (Prevention ,September 1980). Suppose that random samples of 30 adults were selected from two regions of the United States, and a day's intake of selenium ,from both liquids and solids, was recorded for each person. The mean and standard deviation of the selenium daily intakes for the 30 adults from region 1 were 167.1 and 24.3 micrograms respectively. The corresponding statistics for the 30 adults from region 2 were 140.9 and 17.6 micrograms respectively. Find a 95% confidence Interval for the difference in the mean selenium intake for the two regions.