# EASTERN UNIVERSITY, SRI LANKA

## SECOND EXAMINATION IN SCIENCE - 2005/2006

## SECOND SEMESTER (March/April, 2008)

## MT 217 - MATHEMATICAL MODELLING

## (PROPER AND REPEAT)

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Answer all Questions Time: Two hours

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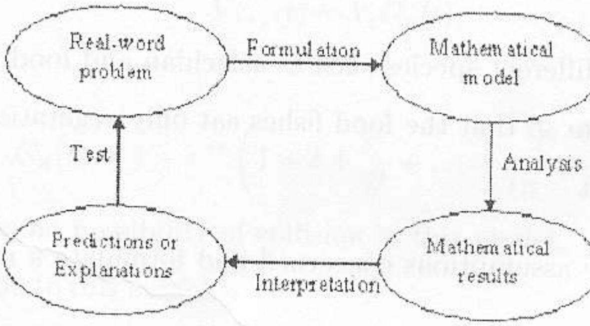
Q1. (a) A schematic flow of modelling process is given below.




Explain the above model building process. [50 marks]

(b) How to categorize the deterministic models?

By considering the type of formulation of the model, type of variables involved and time, explain each of them. [35 marks]

(c) Comment on the following statement.

"Mathematical model building process is an iterative process". [15 marks]

Q2. (a) Define the following terms of a system of interest:

(i) Object;

(ii) Closed and Open system. [30 marks]

(b) In a weight-lifting competition the lifters are grouped into two categories such as Jerk (the weight is lifted first into chest and then to an over-head position) and Snatch (weight is lifted to an over-head position in one motion). Problem involves with the greatest athletic achievement of a group of weight lifters of widely differing weights. The contest is designed as a single competition for all categories of weight lifters and the winner is to be decided using handicapping weight. The goal for the problem is to find a handicapping weight $L'$ to decide the winner when the body weight $W$ of the competitors vary significantly.

(i) What are the major and minor factors affect the weight lifters?

(ii) Discuss the type of formulation of the system.

(iii) If the system is characterized by an assumption that the weight $L$ lifted is proportional to the average cross-sectional area of the lifters muscle, formulate a mathematical model to find the handicapping weight $L'$ as $L' = LW^{-2/3}$. [70 marks]

Q3. Suppose two different species such as selachian and food fishes interact with in the same ecosystem so that the food fishes eat only vegetation and selachian eats only the food fishes.

(a) State the assumptions concerned and formulate a mathematical model in the form of

$$\frac{dx(t)}{dt} = ax(t) - bx(t)y(t), \quad \frac{dy(t)}{dt} = -cy(t) + dx(t)y(t), \qquad (1)$$

where $x(t)$ and $y(t)$ are the population of the food fishes and the selachian, respectively, at time $t$ and $a, b, c$ and $d$ are positive constants. [35 marks]

(b) Find the solution of the mathematical model given by the equation (1) and draw the solution curves in $xy$-plane in which indicate the equilibrium positions. [25 marks]

(c) If $x(t)$ and $y(t)$ are the periodic solutions of the equation (1) with period $T > 0$, by finding the average values $\overline{X}$ and $\overline{Y}$ of $x$ and $y$, respectively, show that the reduced level of fishing is more beneficial to the selachian than to the food fish.

You may use the following relations.

$$\overline{X} = \frac{1}{T}\int_0^T x(t)\,dt, \quad \overline{Y} = \frac{1}{T}\int_T^0 y(t)\,dt.$$



[40 marks]

Q4. Consider $n$ vehicles traveling in a straight line. If $V_n(t)$ is the speed of $n^{th}$ vehicle at time $t$, obtain the model

$$\frac{d}{dt}V_{n+1}(t) = V_n(t) - V_{n+1}(t).$$

Interpret this equation and show that

$$V_{n+1}(t) = \frac{1}{(n-1)!}\int_0^t u^{n-1}e^{-u}V_1(t-u)\,du,$$

where $V_1(t)$ is the speed of the lead vehicle.

Suppose the lead vehicle is standing still at $t = 0$ and acquires a constant cruising speed $V_c$ for $t > 0$, show that

$$V_{n+1}(t) = V_c G_n(t),$$

where

$$G_n(t) = 1 - e^{-t}\left(1 + t + \frac{t^2}{2!} + \ldots + \frac{t^{n-1}}{(n-1)!}\right).$$

Show that there is no possibility of collision in this model.

Give a modification to this model.

[100 marks]