

**EASTERN UNIVERSITY, SRI LANKA**  
**DEPARTMENT OF MATHEMATICS**  
**THIRD EXAMINATION IN SCIENCE -2008/2009**  
**SECOND SEMESTER (Sept./Oct., 2010)**  
**MT 310 - FLUID MECHANICS**  
**(PROPER & REPEAT)**

Answer all Questions

Time: Two hours

1. (a) With the usual notation, derive the continuity equation for a fluid flow in the form

$$\frac{d\rho}{dt} + \rho \nabla \cdot \underline{V} = 0.$$

- (b) In cartesian coordinates, establish the equation of continuity for an incompressible fluid in the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

where  $u, v$  and  $w$  are the cartesian components of the velocity.

Show that

$$u = \frac{-2xyz}{(x^2 + y^2)^2}, \quad v = \frac{(x^2 - y^2)z}{(x^2 + y^2)^2} \quad \text{and} \quad w = \frac{y}{x^2 + y^2}$$

are the velocity components of a possible fluid motion and the motion is irrotational.

- (c) Show that

$$\frac{x^2}{a^2} \tan^2 t + \frac{y^2}{b^2} \cot^2 t = 1,$$

where  $a$  and  $b$  are constants, is a possible form for a boundary surface of a fluid.

2. (a) With the usual notation, derive the *Euler's equation* for an incompressible and inviscid fluid flow.

Hence show that if the fluid flow is steady the *Euler's equation* can be written as

$$(\underline{v} \cdot \nabla)\underline{v} = \frac{F}{\rho} - \frac{1}{\rho}\nabla p.$$

- (b) An incompressible and inviscid fluid obeying Boyle's law  $p = k\rho$ , where  $k$  is a constant, is in motion in a uniform tube of small section. Prove that if  $\rho$  be the density of the fluid then the velocity  $v$  at a distance  $x$  at time  $t$  in the tube is given by the equation

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}[(v^2 + k)\rho].$$

- (c) State the *Kelvin circulation* theorem.

If the velocity field is given by  $\underline{v} = \frac{-y\hat{i} + x\hat{j}}{x^2 + y^2}$  then calculate the circulation around a square with its corners at  $(1, 0), (2, 0), (2, 1), (1, 1)$ .

3. (a) Let a gas occupy the region  $r \leq R$ , where  $R$  is a function of time  $t$ , and a liquid of constant density  $\rho$  lie outside the gas. By assuming that there is contact between the gas and the liquid all the time and that the motion is symmetric about the origin  $r = 0$ , show that the motion is irrotational.

If the velocity at  $r = R$ , the gas liquid boundary is continuous then show that the pressure  $p$  at a point  $P(\underline{r}, t)$  in the liquid is given by

$$\frac{p}{\rho} + \frac{1}{2} \left( \frac{R^2 \dot{R}}{r^2} \right)^2 - \frac{1}{r} \frac{d}{dt}(R^2 \dot{R}) = f(t),$$

where  $|\underline{r}|=r$  and dots denote differentiation with respect to time  $t$ .

- (b) Given that a liquid extends to infinity and is at rest there with constant pressure  $\pi$ . Prove that the gas and liquid interface pressure for a spherical bubble of radius  $R$  is

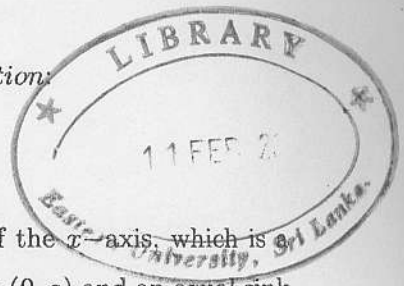
$$\pi + \frac{\rho}{2R^2} \frac{d}{dR}(R^3 \dot{R}^2).$$

If the gas obeys Boyle's law  $p v^{1+\alpha} = \text{constant}$ , (where  $\alpha$  is a constant and  $v$  is the volume of the gas) and expands from rest at  $R = a$  to a position of rest at  $R = 2a$ , deduce that the initial pressure is

$$\frac{7\alpha\pi}{1 - 2^{-3\alpha}}.$$

4. (a) With the usual notation, derive the *Bernoulli's equation*:

$$\int \frac{dp}{\rho} + \frac{1}{2}v^2 + \Omega = \text{constant.}$$



- (b) If fluid fills the region of space on the positive side of the  $x$ -axis, which is a rigid boundary and if there be a source  $m$  at the point  $(0, a)$  and an equal sink at  $(0, b)$  and if the pressure on the negative side be the same as the pressure at infinity, show that the resultant pressure on the boundary is

$$\frac{\pi \rho m^2 (a - b)^2}{2ab(a + b)},$$

where  $a > b$ ,  $\rho$  is the density of the fluid and  $\pi$  is the pressure of the fluid.