EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
THIRD EXAMINATION IN SCIENCE -2008/2009
SECOND SEMESTER (Sept./Oct., 2010)
MT 310 - FLUID MECHANICS
(PROPER & REPEAT)

Answer all Questions **Time: Two hours**

1. (a) With the usual notation, derive the continuity equation for a fluid flow in the form

$$\frac{d\rho}{dt} + \rho \underline{\nabla} \cdot \underline{V} = 0.$$

(b) In cartesian coordinates, establish the equation of continuity for an incompreessible fluid in the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

where u, v and w are the cartesian components of the velocity.

Show that

$$u = \frac{-2xyz}{(x^2+y^2)^2}, \quad v = \frac{(x^2-y^2)z}{(x^2+y^2)^2} \text{ and } w = \frac{y}{x^2+y^2}$$

are the velocity components of a possible fluid motion and the motion is irrotational.

(c) Show that

$$\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 1,$$

where a and b are constants, is a possible form for a boundary surface of a fluid.

2. (a) With the usual notation, derive the *Euler's equation* for an incompressible and inviscid fluid flow.

Hence show that if the fluid flow is steady the *Euler's equation* can be written as

$$(\underline{v}\cdot\underline{\nabla})\underline{v} = \underline{F} - \frac{1}{\rho}\underline{\nabla}p.$$

(b) An incompressible and inviscid fluid obeying Boyle's law $p = k\rho$, where k is a constant, is in motion in a uniform tube of small section. Prove that if ρ be the density of the fluid then the velocity v at a distance x at time t in the tube is given by the equation

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}[(v^2 + k)\rho].$$

(c) State the *Kelvin circulation* theorem.

If the velocity field is given by $\underline{v} = \dfrac{-y\underline{i} + x\underline{j}}{x^2 + y^2}$ then calculate the circulation around a square with its corners at $(1,0), (2,0), (2,1), (1,1)$.

3. (a) Let a gas occupy the region $r \leq R$, where R is a function of time t, and a liquid of constant density ρ lie outside the gas. By assuming that there is contact between the gas and the liquid all the time and that the motion is symmetric about the origin $r = 0$, show that the motion is irrotational.

If the velocity at $r = R$, the gas liquid boundary is continuous then show that the pressure p at a point $P(\underline{r}, t)$ in the liquid is given by

$$\frac{p}{\rho} + \frac{1}{2}\left(\frac{R^2\dot{R}}{r^2}\right)^2 - \frac{1}{r}\frac{d}{dt}(R^2\dot{R}) = f(t),$$

where $|\underline{r}|$=r and dots denote differentiation with respect to time t.

(b) Given that a liquid extends to infinity and is at rest there with constant pressure π. Prove that the gas and liquid interface pressure for a spherical bubble of radius R is

$$\pi + \frac{\rho}{2R^2}\,\frac{d}{dR}(R^3\dot{R}^2).$$

If the gas obeys Boyle's law $pv^{1+\alpha}$ =constant, (where α is a constant and v is the volume of the gas) and expands from rest at $R = a$ to a position of rest at $R = 2a$, deduce that the initial pressure is

$$\frac{7\alpha\pi}{1 - 2^{-3\alpha}}.$$

4. (a) With the usual notation, derive the *Bernoulli's equation*:

$$\int \frac{dp}{\rho} + \frac{1}{2}v^2 + \Omega = \text{constant}.$$

(b) If fluid fills the region of space on the positive side of the x-axis, which is a rigid boundary and if there be a source m at the point $(0, a)$ and an equal sink at $(0, b)$ and if the pressure on the negative side be the same as the pressure at infinity, show that the resultant pressure on the boundary is

$$\frac{\pi \rho m^2 (a-b)^2}{2ab(a+b)},$$

where $a > b$, ρ is the density of the fluid and π is the pressure of the fluid.